Demand driven ecological collapse:

A stock-flow fund-service model of money, energy and ecological scale

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The python code for the simulations is available at https://oliver-richters.de/sfc-models/barth-richters-ecological-collapse-model-2019.py.

1 Introduction

One of the key issues faced by modern society is navigating the transformation towards a sustainable economy that respects 'planetary boundaries' (Rockström *et al.*, 2009). However, most models in the line of neoclassical theory taught to students worldwide are neglecting environmental and physical variables. In addition, most general equilibrium models abstract from the monetary stocks and flows and base their reasoning on 'real' economic variables and exchange. The word 'real' does in no way mean that 'physical' variables such as energy or material flows are treated. This chapter highlights possible links between ecological and post-Keynesian economics and develops a simple toy model that can serve as foundation for studying interrelations between the monetary economy and the physical environment. Its economy is modeled in discrete time using a balance sheet approach, and production is demand driven. The source of production is the ecosystem that is affected by economic 'harvest', and economic degradation can generate restrictions on supply. Both the monetary and physical economy are modeled in a stock-flow consistent way.

The chapter is structured as follows: In the next section, we give a short overview about the theoretical background. We outline important aspects of post-Keynesian monetary theory and ecological economics considering a possible synthesis. In the second chapter, we provide a comprehensive description of what we call the monetary physical stock-flow fund-service model. First simulation results provide a first intuition of the model behavior. A stability analysis gives general insights into the parameter dependence of different model outcomes. The chapter is completed with final remarks.

2 Theoretical Background

2.1 Monetary theory of production

Most neoclassical theories tend to assume that money is neutral in the long term, a mere numeraire or means of exchange, without significant differences from circulating commodities. It improves efficiency of barter but plays a rather passive role in the economic process. Therefore, the impact of monetary issues on long-run economic processes such as economic growth or environmental issues is considered negligible. Those emphasizing the need for a monetary theory of production reject this assumption. One effort to explicitly represent the dynamics of debt, finance, and other monetary factors has been the balance sheet approach (see the chapter by Dirk Ehnts in this book), that tracks assets and liabilities and their interdependence in the economy. The central importance of attention to financial detail was illustrated by the failure of the macroeconomics profession to anticipate the 2007–2008 Global Financial Crisis, which was predicted nearly exclusively by those who deployed implicit or explicit macro-accounting frameworks (Galbraith, 2009; Bezemer, 2010; Koo, 2011).

In recent years, models based on these accounting relationships and a coherent study of monetary stocks and flows were advanced particularly by post-Keynesian authors using the term 'stock-flow consistent models' (SFC), see Godley and Lavoie (2012). This strand of theory argues with reference to Keynes (1936) that production adapts to demand. Keynesian macroeconomic theory places great emphasis on the determination of a level of effective demand commensurate with key economic policy goals, but the ecological implications of those economic policy goals have often been neglected (Berg *et al.*, 2015).

2.2 Ecological Economics

Ecological economists have criticized approaches such as SFC models on grounds that they focus on the circular flow of exchange value (i.e. money), rather than on the physical throughput of natural resources from which all goods and services are ultimately derived (Georgescu-Roegen, 1971; Daly, 1985). Sustainable economic activity that 'meets the needs of the present without compromising the ability of future generations to meet their own needs' (WCED, 1987) has to stay within an environmentally sustainable scale: the ecosystem has to absorb waste and recycle the inputs which are required for physical production (Daly, 1992). As capital is highly dependent on energy usage, the regeneration rate and the availability of renewable energy resources are the final constraints to the production process (Dale *et al.*, 2012). However, the importance of energy and natural resources for economic production is systematically underestimated in many economic theories (Kümmel, 2011). Thus, analyzing the physical and environmental sustainability requires studying the interdependencies between the ecosystem and the economy. Georgescu-Roegen (1971) has emphasized that models need to track the physical funds and flows of physical variables such as energy explicitly. Within ecological economics, monetary questions are studied only recently (Berg *et al.*, 2015; Dafermos *et al.*, 2017).

2.3 Common ground: Ecological Macroeconomics

From a philosophy of science perspective, synthesis of different economic paradigms seems possible, if these share similar ontological and methodological approaches (Dobusch and Kapeller, 2012). Several authors have explicitly argued that post-Keynesian economics and Ecological economics share substantial common ground, and are ripe for a synthesis. Ontological similarities have been recognized in terms of consumption, production theory, cumulative causation (path dependency), uncertainty as opposed to computable probability and the irreversibility of historical time (Gowdy, 1991; Lavoie, 2006; Holt *et al.*, 2009; Kronenberg, 2010). Post-Keynesians argue that this uncertainty and instability is inherent in economic processes while ecological economists locate the reason within environmental risks. Therefore, intertemporal optimization with unlimited time horizon and rational expectations about the future seems unrealistic. Ecological economists arguing that consumption is a central driver of economic growth also seem to agree with the Keynesian argument that effective demand matters.

From a methodological point of view, there are similarities in looking at the world as composed of a complex system of stocks and flows, ecological economists mostly from a physical perspective and post-Keynesian authors from a monetary perspective. This lends itself for economic modelling and is therefore suitable for bridging the gap between ecological and post-Keynesian models. Models that integrate monetary and ecological issues may be helpful to study pressing problems such as climate change, which are neither purely economic, nor purely environmental, nor purely physical, but rather are all the above. The recent development of 'ecological macroeconomics' has developed exploiting these similarities (Berg *et al.*, 2015; Rezai and Stagl, 2016), for example for studying the stability of a non-growing economy (Richters and Siemoneit, 2017).

The only approach known to us to integrate the physical framework by Georgescu-Roegen and the monetary stock-flow consistent framework is Dafermos *et al.* (2017). Different to their very complex and interdependent model, we offer a simple toy model that may help to understand and combine the two paradigms. The model treats physical and monetary stocks and flows explicitly and is coupled to a minimal environmental model.

2.4 The stock-flow fund-service approach

Before we get to the description of the model, we shortly want to outline the idea of the stock-flow fund-service approach (SFFS), which provides a common framework for model development.

Stocks represent an amount of energy, matter or money at a point of time given in J, kg or €, whereas flows represent a stream of matter, energy or money from one stock to another in a certain period of time, given or J/s, kg/s or €/s. The distinctive feature is, that stocks can be instantaneously consumed or transferred as a whole (Georgescu-Roegen, 1971). This concept is also used in post-Keynesian SFC models, where the balance sheets include the financial stocks and the transaction matrix the financial flows from one sector to another (Godley and Lavoie, 2012), which we will describe below.

Besides the stock-flow approach it is useful to introduce another concept, as proposed by Georgescu-Roegen (1971, p. 224 ff.) – the fund-service approach, which is not included in the post-Keynesian theory. A fund represents the counterpart to stock/flows. They cannot be instantaneously consumed, such as the service of the worker to assemble a good (Georgescu-Roegen, 1971, p. 226) or the sun to provide high-energetic radiation. One square meter of land does only receive a certain maximum amount of 'radiation service' at any moment in time. Therefore, the amount of funds includes the time dimension ($service \cdot time$) whereas the service is without a time dimension. Note that this is the other way around for stocks and flows.[1]

Here we also find the impossibility of complete substitutability, which is often criticized by ecological and post-Keynesian economists likewise (Kronenberg, 2010). Stocks and funds are qualitatively different. So, one cannot be replaced by the other. If one needs a flow as energy, e.g. for production, one can have as much capital as she wants. If there is no energy left for consumption, there is no possibility to substitute it by something else. This is reflected in the concept of strong sustainability (Ott and Döring, 2008), as counterpart to the weak formulation, which is mostly used by environmental economists such as Perman (2011).

1 In the theory of monetary economics, the term fund is used in the label "flow of funds". However, this represents financial transfers. Note that the meaning used here differs from this definition.

3 Description of the monetary-physical Stock-flow fund-service model

In the following we describe the SFFS model for a one good economy which includes physical services as well as energy and monetary flows. The model represents a dynamical system in discrete time. Both monetary and physical flows are equally represented. Its monetary representation is based on the model SIM of Godley and Lavoie (2012), whereas money is replaced by government bills and interest payments are included in addition.[2]

For a better understanding, we differentiate physical flows with upper index p from monetary flows without an upper index. The former is measured in Joule J per time step as measure for energy, the latter in € per time step as measure for money. The monetary flows are described in Table 1, physical flows and services in Table 2. Both are represented in Figure 1. In the tables, the direction of the flows is indicated by the signs. Positive values are uses of the stock and represent outflows, while negative values are sources and represent inflows accordingly. Time is indicated by the index t, whereas t $(t-1)$ indicates the stock/flow at the end of the current (previous) period. In the diagram, the different sectors described by their balance sheets are connected by monetary flows depicted with solid arrows. Flows of physical goods are dashed, while the transfer of energy is indicated by broad, filled arrows.

The economy consists of households, one production sector and one government sector[3]. Consumption expenditures C flow from households to the production sector. In this sector, products are produced and the incoming monetary flows from consumption are fully paid as wages Y to the households[4]. For simplicity government engages workers directly for governmental duties.

Besides of consumption, households use their income from wages, government expenditures and interest on government bills for paying taxes T to the government. All remaining income is accumulated as government bills ΔM_h.[5] Accordingly, the government receives taxes, pays interest on government bills, expenditures to workers and the amount of bills issued by the government increases by ΔM_g.

The system is closed "in the sense that everything comes from somewhere and everything goes somewhere," with this framework, "there are no black holes" (Godley and Lavoie, 2012, p. 38). Also, the balance sheet – the stocks

2 Bills are a short-term debt obligation backed by the government with a maturity of less than one year, here: in one period.

3 SFC model with various sectors, banks and different financial assets can be found in Godley and Lavoie (2012) or Berg *et al.* (2015).

4 For the sake of simplicity we neglect the differentiation between profits and wages.

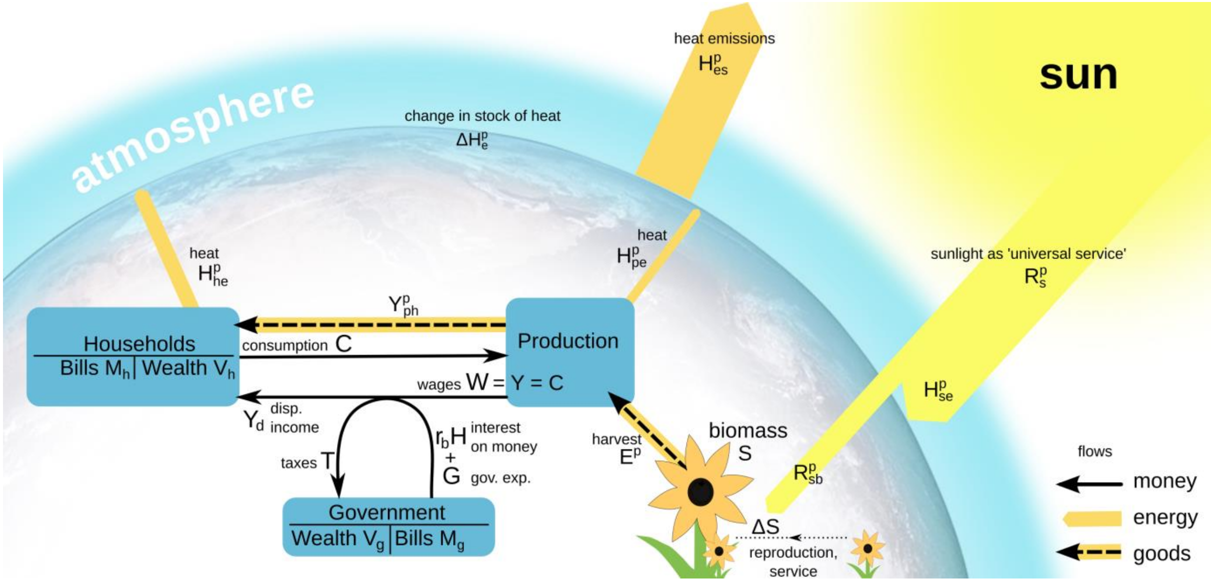
5 It is often noted, that sign for the change in financial assets is counterintuitive, because the acquisition of government bills seems to be an incoming flow resulting in a positive sign. However, this must be interpreted as households paying from the money they earn into their stock of government bills. Even if this is an action happening within the household it is a use of money which consequently carries a negative sign (Godley und Lavoie 2012, S. 40).

– as provided in Figure 1 cancel out as the bills held by households M_h must always be equal to the bills issued by the government M_g.

Table 1: Monetary transaction-flows matrix

Monetary Transaction matrix				
	Households	Production	Government	$\sum$
Consumption	$-C$	$+C$		0
Government exp.	$+G$		$-G$	0
Wages	$+Y$	$-Y$		0
Taxes	$-T$		$+T$	0
Interest Payments	$+rM_{t-1}$		$-rM_{t-1}$	0
Change in bills	$-\Delta M_h$		$+\Delta M_g$	0
$\sum$	0	0	0	0

Figure 1: Monetary Stocks/flows and physical funds/services of the model



The monetary economy is embedded into an ecological system, as shown in Figure 1. The interaction between environment and economy occurs by harvesting biomass to provide low entropy energy (exergy) input for the production of the economy E^p. Within the production sector this biomass is transformed into an energy good such as oil. The biomass can be thought of as sunflowers, corn, or similar plants which are suitable for transformation into energy inputs for production and for consumption likewise and are renewable.

The good created is used for production itself Y^p_{pp} or transferred to the household Y^p_{ph}. There it is finally consumed (C^p) and ends up as heat (anergy) on planet Earth H^p_{pe}. Same is true for the consumption within the production process H^p_{pe}. It becomes obvious that the economic subsystem is fully dependent on harvest, without which no production is possible. However, the reproduction of biomass ΔS is an environmental service and not a flow and therefore does not happen instantaneous but needs time, as shown in 2.4. The service increases the stock of biomass, from which harvest is subtracted. Consequently, the rate of harvest at maximum has to equal the rate of renewal for the system to be ecologically sustainable.

Table 2: Physical transaction flow-service matrix

	Households	Production	Biomass, Low entropy	Heat on Earth High entropy	Space	Σ
Flows						
Household consumption	$-C^p$			$+H^p_{he}$		0
Product transfer	$+Y^p_{ph}$	$-Y^p_{ph}$				0
Energy consumption		$-Y^p_{pp}$		$+H^p_{pe}$		0
Harvest		$+E^p$	$-E^p$			0
Dissipation				$-H^p_{es}$	$+H^p_s$	0
Services						
Change in stock of Energy			$-\Delta S$ (biomass)	$-\Delta H^p_e$	$-\Delta H^p_s$	0
Radiation and absorption			$+R^p_{sb}$	$+H^p_{se}$	$-R^p_s$	0
Σ	0	0	0	0	0	0

For the sake of completeness in Table 2 we also included low and high entropy energy streams from space to the Earth, to have a realistic energy balance. In doing so, the physical system is closed in the sense that everything that goes somewhere comes from somewhere. A constant inflow of exergy enters the system as sunlight (R^p_s), some parts are transformed to biomass (R^p_{sp}) and changes, together with harvest (E^p) the stock of biomass (ΔS). The radiation that is not metabolized is dissipated on the Earth' surface and in the atmosphere (H^p_{se}). Harvest (E^p) and consumption of biomass by households and firms turn the stored energy into anergy (H^p_{pe}, H^p_{he}) and reduce the stock of biomass. A constant flow of anergy leaves the Earth as radiation to space (H^p_{es}). To determine the distribution of energy changes to space and Earth ($\Delta \mathrm{H}^{\mathrm{p}}_{\mathrm{e}},\ \Delta \mathrm{H}^{\mathrm{p}}_{\mathrm{s}}$), a climate model would be needed.

One can easily see the irreversibility of the energy consumption process here. As Georgescu-Roegen (1971, p. 281) puts it: "[The economic process] is not circular, but unidirectional. As far as this facet alone is concerned, the economic process consists of a continuous transformation of low entropy into high entropy, that is, into irrevocable waste or, with topical term, into pollution." (Georgescu-Roegen, 1971, p. 281). Pollution, also referred as heat emissions or anergy, can neither be used for the reproduction of biomass nor the production process. Rather all exergy consumed finally ends up as heat in on Earth. For climate models these anthropogenic heat emissions are estimated to be about 0.0025 Wm^{-2} (Stocker et al., 2013, p. 14). Besides of the ecological constraints given by the reproduction of biomass, we face another hidden constraint here. If the amount of heat emissions increases to exceed $3 \cdot 10^{14}$ W the economy crosses the so called “heat barrier” (Buttlar, 1975), where global climate changes will occur even without the anthropogenic greenhouse effect (Kümmel, 2011, p. 179). This could be included using for example a simple climate model as provided in Berg *et al.* (2015).

For the sake of simplicity, we will refer to the left part of Table 2 framed in black in the remainder of this paper and leave the rest open for further research. In the following we will describe the balance equations and the behavior of the system.

3.1 Monetary behavioral equations

The model is consumption driven, meaning that the level of consumption of households C_t determines the level of production Y_t.

$$Y_{ph,t} = C_t. \quad (1)$$

To simplify the model, we assume that government expenditures $G_t = G$ are constant over time and go directly to the households. All earnings by the production sector C_t are paid to households as wages. Capital is not included in the model.

Additionally to wages, households receive income by interest payments with interest rate r on bills M_t from the government sector. r is assumed to be exogenously given and constant. Taxes with rate θ are paid on income, so disposable income $Y_{D,t}$ and taxes T_t are given by

$$Y_{D,t} = (1-\theta)(C_t + G_t + rM_t), \quad (2)$$

$$T_t = \theta(C_t + G_t + rM_t). \quad (3)$$

The discrete equation of motion for the stock of bills ($M_h$and $M_g$that are identical) is given by

$$M_t = M_{t-1} + Y_{D,t} - C_t. \quad (4)$$

Bills of the previous period M_{t-1} is increased by disposable income less realized consumption C_t. Note that households have a consumption target C_t^T, which is different to realized consumption C_t. It is determined by the propensity to consume out of disposable income of the last period (the first term) and out of wealth (the second term)[6]

$$C_t^T = c_y(1-\theta)(C_{t-1} + G_{t-1} + rM_{t-1}) + c_M M_{t-1}. \quad (5)$$

We differentiate between targeted and realized consumption C_t because targeted consumption may be decreased due to ecological constraints, which will be the focus of the physical equations (s. Equation (11)). For targeted production, it follows

$$Y_{ph,t}^T = C_t^T. \quad (6)$$

3.2 Physical flow equations

Before producers can produce anything, there must be a stock of biomass. We differ here from Heyes (2000) and Lawn (2003) and assume that biomass is not growing exponentially if undisturbed, but as logistic growth. These growth functions are very common in environmental modelling (Wainwright and Mulligan, 2013).

$$\mathrm{S_t} = \mathrm{S_{t-1}} + \mathrm{S_{t-1}a}\left(1 - \frac{\mathrm{S_{t-1}}}{\mathrm{S_{max}}}\right) - \mathrm{E_t} \text{ with } 0 < \mathrm{a} < 2.6 \quad (7)$$

The growth function can be thought of as a S-curve, whereas the stock is limited to $\mathrm{S_{max}}$. For small $\mathrm{S_{t-1}}$ the stock of the next period $\mathrm{S_t}$ does only increase exponentially, for $\mathrm{S_{t-1}} = \frac{1}{2}\mathrm{S_{max}}$ absolute growth is maximum with $\Delta\mathrm{S} = \frac{1}{4}\mathrm{aS_{max}}$ to decline as $\mathrm{S_{t-1}}$ approaches $\mathrm{S_{max}}$, as shown in Figure 2. Thus the relation between the radiation turned into biomass $\mathrm{R_{sb}^p} = S_{t-1}a\left(1 - \frac{S_{t-1}}{S_{max}}\right)$ and the dissipation to heat on Earth $\mathrm{H_{se}^p}$ is altered depending on the biomass already available: In the desert, all radiation is turned to heat, as no radiation is transformed into biomass. The

6 Consumption out of interest payments is neglected to simplify the model

maximum absolute growth corresponds to the optimal use of radiation input, thus $\frac{1}{4}\mathrm{aS_{max}} = \mu\mathrm{R_s^p}$ with $\mu < 1$ because of sunlight reflection and efficiency of photosynthesis.

Physical and monetary flows are linked by the fixed parameter price $\mathrm{p} \in \frac{€}{\mathrm{J}}$ to physical flows.

$$C_t = p \cdot C_t^p; \quad C_t^T = p \cdot C_t^{T,p}; \quad Y_{ph,t}^T = p \cdot Y_{ph,t}^{T,p} \tag{8}$$

Price adaptations play a crucial role in many dynamic models. Nevertheless, we chose to treat prices as constant parameters in our model to avoid a detailed discussion of the strategic determination of prices and the conception of the firm in post-Keynesian theory, and because the added dimension would have made the stringent stability analysis less comprehensible. The inclusion of price adaptation may significantly alter the results.

Total targeted production is determined by consumption and resulting own energy requirements of the production sector $Y_{pp,t}^p$. We assume the requirements of energy being proportional to production with $0 < \epsilon < 1$.

$$Y_t^{T,p} = Y_{pp,t}^p + Y_{ph,t}^{T,p} = (1+\epsilon)C_t^{T,p}. \tag{9}$$

The stock of biomass is decreased by harvest E_t in kg which satisfies the need for biomass of production for total production Y_t^p. To account for different units $\gamma \left[\frac{kg}{J}\right]$ is introduced to transfer the stock of biomass, given in kg, to energy units.

$$E_t = \gamma Y_t^p \tag{10}$$

To include the ecological limits on harvest which are given by the stock of biomass S_t at period t the following equation constrains total targeted harvest $E_t^T = Y_t^{T,p}$including energy requirements by the production sector to realized production E_t^p.

$$E_t = \frac{E_t^T}{1 + E_t^T / S_{t-1}} = \frac{\gamma(1+\epsilon)C_t^T}{p\,(1 + \gamma\,(1+\epsilon)C_t^T / (p \cdot S_{t-1}))} \tag{11}$$

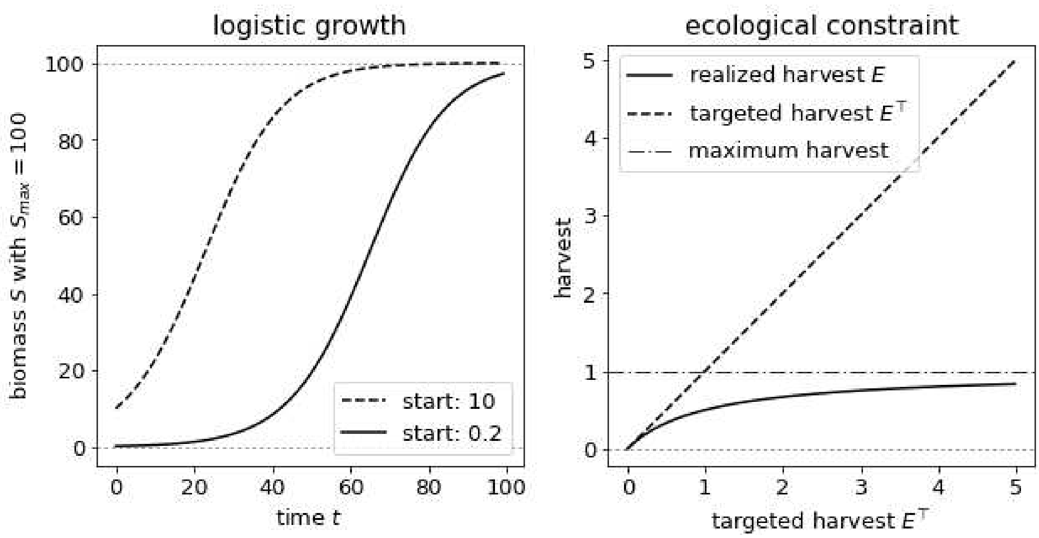


Figure 2: Example of logistic growth for different initial values and maximum growth rate $a = 0.1$ (left) and constrained production (right) following Equation (11) with $S_{t-1} = 1$.

The rationale behind Equation (11) is that if harvest is small compared to the stock of biomass (as is for example the case for forestry in most countries), the realized harvest is close to identical to the desired one, to demand. On the other hand, if targeted harvest is very high, the dynamical system has to guarantee that the harvest does not exceed the available stock, which is impossible. This equation is an example how such a 'smooth rationing' in the case of ecological scarcity can be implemented. Compared to a piecewise linear function, it avoids discontinuities which would make the stability analysis much more challenging.

It follows for the realized physical consumption of households with Equation (8) is given by

$$C_t^p = \frac{Y_t^p}{1+\epsilon}. \tag{12}$$

3.3 System of equations

Accordingly, we can derive the equations that determine the behavior of the system. To simplify the notation, we use the relation $\gamma_\epsilon = (1+\epsilon)\gamma$ and $Y_t^p = \frac{Y_{t,\epsilon}^p}{1+\epsilon}$. Y_t^p differs from $Y_{t,\epsilon}^p$ by not including internal energy consumption of production.

$$Y_t^p = \frac{C_t^T}{p + \gamma_\epsilon C_t^T / S_{t-1}} \tag{13}$$

$$S_t = S_{t-1} + S_{t-1} a \left(1 - \frac{S_{t-1}}{S_{max}}\right) - \gamma_\epsilon Y_t^p with 0 < a < 2.6 \tag{14}$$

$$M_t = M_{t-1} + (1-\theta)(G_0 + rM_{t-1}) - \theta Y_t^p p \tag{15}$$

With

$$C_t^T = c_y(1-\theta)\left(pY_{t-1}^p + G_0 + rM_{t-1}\right) + c_M M_{t-1} \tag{16}$$

$$Y_{t,\epsilon}^p = Y_t^p(1+\epsilon) \tag{17}$$

It is a three-dimensional system depended on M_t, Y_t^p and S_t which determines the behavior of the system. Equations (16) and (17) are only substitutes for the values of C_t^T and $Y_{t,\epsilon}^p$, so the system remains three-dimensional. For the simulation, one has to know the initial values of Y_0^p, M_0 and S_0. With these initial conditions we can calculate C_t^T according to Equation (16) by using values known from the previous period C_{t-1} and M_{t-1}. The result is substituted into Y_t^p from Equation (13) which goes into S_t via Equation (14) and M_t via Equation (15). This procedure is repeated for as many time steps as preferred.

4 Simulation results of the model

To give a first intuition of the model behavior, we obtained simulation results for the model with Python. From these, we can derive three different kinds of model behavior, represented in Figure **3**. The left graph shows an ecologically and monetarily stable system that converges towards a stationary state for M, S, and Y. In this case economically driven harvest and ecologically driven reproduction are equal leading to an ecological stability. Furthermore, the system is monetarily stable, due to high levels of consumption out of wealth compared to the interest rate. We will give a detailed explanation of this behavior in Section 5. In the middle graph, the system tends to a monetarily stable stationary state, indicated by the concave shape of M and Y. However, as one can see from the monotonically declining course of S the ecological system is continuously overused leading to an ecological collapse at $t = 360$ causing an immediate economic collapse. However, since the government continues to pay interests on bills the monetary stock continuously increases. A more drastic result is obtained in the 'explosive' right graph of Figure **3**. Due to high interest rates compared to consumption out of wealth, the system is monetarily unstable and grows out of bound, indicated by the convex shape of M. As the economy cannot grow forever due to ecological constraints, it collapses at $t = 150$ – just like in the middle graph – because it continuously overuses the ecology. Note that in this scenario, the government debt to GDP ratio increases unlimitedly even before the ecological collapse – thus we don't see stable growth but rather a debt spiral where the consumptive government expenditures become negligible compared to debt services, not indicating economic stability. Furthermore, the model abstracts from expectations that might cause negative output effects due to high debt to GDP ratios.

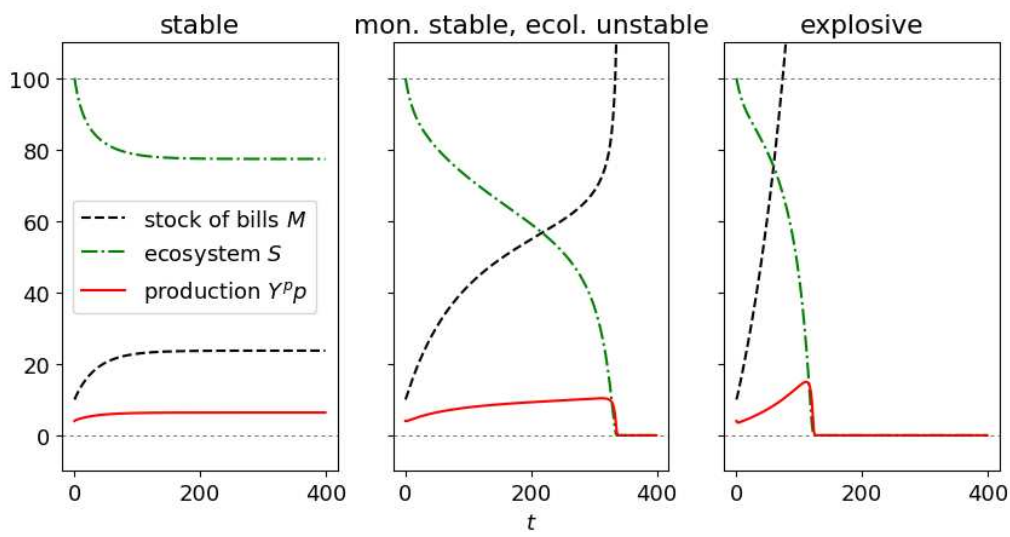


Figure 3: Time evolution of the system for different propensities to consume out of wealth. Parameters $\theta = 0.5, c_y = 0.8, a = 0.1, p = 4, \gamma = 1.1, S_{max} = 100, G = 4, r = 0.1$ and initial values $S_0 = 100$, $Y_0^p = 1, Y_0 = pY_0^p = 4, M_0 = 10$. Left graph: monetarily and ecologically stable system with $c_m = 0.06$. Center: monetarily stable, but ecologically unstable economy with $c_m = 0.04$. Right graph: monetarily and ecologically unstable 'explosive' system with $c_m = 0.01$.

5 Stability Analysis of the model

To analyze the model, explain the differences pictured in Figure 3 and derive more insights in its general behavior, we conducted a stability analysis of the three-dimensional system.[7] For the calculation of fixed points, where no change in either variable occurs, we must state that

$$S_t = S_{t-1}; \quad M_t = M_{t-1}; \quad Y_t^p = Y_{t-1}^p. \tag{18}$$

In doing so, we can derive the coordinates of the fixed points (see Appendix A). The stationary state for the biomass stock S^* is the closed form solution of the cubic equation

$$\frac{a}{\gamma_\epsilon} S^* \left(1 - \frac{S^*}{S_{max}}\right) \underbrace{\left(\frac{1}{1 - a\left(1 - \frac{S^*}{S_{max}}\right)} - c_y - \frac{c_m \theta}{(1-\theta) r}\right)}_{F(S)} = \frac{-c_m}{rp} G. \tag{19}$$

Reasonable results can be obtained for $S^* \in [0, S_{max}]$, whereby the equation is solvable for $r \neq 0; \theta \neq 1; p \neq 0; a \neq 1; \gamma_\epsilon \neq 0$.

Knowing S^* we can calculate the stationary states of production Y^{p*} and the stock of bills M^* with

$$Y^{p*} = \frac{a}{\gamma_\epsilon} S^* \left(1 - \frac{S^*}{S_{max}}\right), \tag{20}$$

$$M^* = \frac{\theta p Y^{p*} - (1-\theta) G}{1 - \theta r}. \tag{21}$$

From Equations (19) to (21) we can conclude, that the number of fixed points depends solely on the number of solutions for Equation (19). Since by definition $S^* \in [0, S_{max}]$ we can state further, that no solution exists and accordingly, there will be no stationary state within this domain, if $F(S^*) > 0$. In this case, the stock of biomass $S^* < 0$ or $S^* > S_{max}$. We can interpret this result as a global instability, which leads to over-depletion of the biomass stock and consequently to the collapse of the ecological system. As shown in Appendix B $F(S^*) > 0$ for any $S^* \in [0, S_{max}]$if

$$c_r = \frac{c_m}{r} < \frac{(1 - c_y)(1-\theta)}{\theta}. \tag{22}$$

This result is equal to the relation derived for the SIM model of Godley and Lavoie (2012) by Richters and Siemoneit (2017). However, in contrast to this paper, they do not consider ecological variables. Note, that this relation is independent of the biomass growth rate a. Therefore, it can be interpreted as the *monetary stability condition*. If Inequality (22 is fulfilled, consumption and production will increase unboundedly which necessarily leads to an ecological collapse at some point in time in our model, even for very high ecological regeneration. However, it is possible to have positive interest rates within a monetarily stable economy. In this case, positive interest rates

7 For general information on stability analysis see Argyris *et al.* (2015). An application to stock-flow consistent models is provided by Richters and Siemoneit (2017).

have a positive effect on the stock of government bills of the economy, as they determine the speed of the accumulation of bills. If the dampening effect by consumption out of wealth c_m is high compared to the interest rate r the economy becomes stable (compare Figure 1).

To analyze the conditions for stability dependent on changes on certain parameters, as for example the ratio $c_r$of c_m and r, we can convert Equation (19) to an implicit function as follows:

$$K(S^*,c_r) := \frac{a}{\gamma_\epsilon} S^*\left(1-\frac{S^*}{S_{max}}\right)\left(\frac{1}{1-a\left(1-\frac{S^*}{S_{max}}\right)} - c_y - \frac{c_r\theta}{(1-\theta)}\right) + \frac{c_r}{p}G = 0. \tag{23}$$

We can now picture the solutions for Equation (19) for different values of c_r by plotting the implicit function of Equation (47), as shown in the bifurcation diagram in Figure **4**.

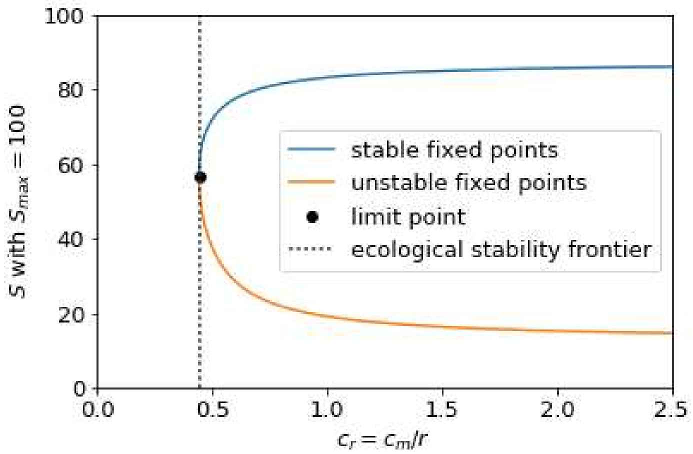


Figure 4: Bifurcation diagram for the stationary state of the biomass stock S and the consumption out of wealth - interest rate ratio c_r. Parameter values: $\theta = 0.5, c_y = 0.8, a = 0.1, G/p = 1, \gamma_\epsilon = 1.1, S_{max} = 100$.

On the x axis, we see $c_r = \frac{c_m}{r}$ and on the y-axis the solutions S^*, which are the fixed points for any value of c_r with θ,a, p, G, γ_ϵ, S_{max} and c_y as fixed parameters. For $c_y < 0.44$, no fixed point exists and the system becomes unstable. We can calculate this value by solving Equation (47) for $c_r = H(S^*)$ (see Appendix D) and deriving the minimum value for c_r from $H'(S^*) = \frac{dH(S^*)}{dS^*} = 0$ in the domain $S^* \in [0, S_{max}]$. In general, we can derive the *ecological stability condition*

$$\nexists S^* \in [0, S_{max}]\ if\ c_r < min\big(H(S^*_{min})\big). \tag{24}$$

For $c_r > 0.44$ the lower fixed point is unstable. Consequently, if the initial value for S_0 is below the orange line in Figure **4** the system will not converge towards the stable fixed point, but collapse. If the initial condition is above the orange line, the system will converge towards the stable fixed point (blue line). Another way to obtain these conditions is by solving the Jacobian matrix at a fixed point S^*, which is derived and presented in Appendix C, and calculating the eigenvalues of the matrix. For $c_r = 1$ the eigenvalues are derived for the fixed point below the limit point ($S^*_L = 19.1$) in Figure **4**, and for the fixed point above the limit Point ($S^*_H = 83.1$):

$$\lambda_{S^*_L} = (0.37 \quad 0.95 \quad 1.06), \lambda_{S^*_H} = (0.43 \quad 0.93 \quad 0.94).$$

Since one of the eigenvalues of S_L^* is higher than one, this fixed point is unstable. For S_H^* all eigenvalues are lower than one, consequently this fixed point is stable and the system will converge to it.

From the conditions in Equation (22) and (24) we can now plot a stability diagram (Figure 5), which shows the existence of stationary states at different values of c_m and r.

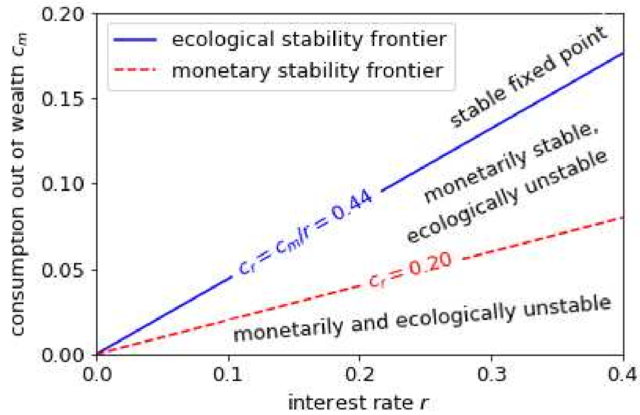


Figure 5: Stability diagram for the ecological stability condition and the monetary stability condition for different values of consumption out of wealth $c_m$and interest rates r with $\theta = 0.5, c_y = 0.8, a = 0.1, \gamma_\epsilon = 1.1, S_{max} = 100, G/p = 1$.

This result adds further insights to the results derived by Richters and Siemoneit (2017) by adding an ecological dimension. If the ratio of c_m and r is above (below) the monetary stability condition (dashed line) the system is monetarily stable (unstable). This is equivalent to the results of the middle and right graph in Figure 3. Without ecological constraints, the stock of bills and the level of production would converge towards stationary states M and Y^p. However, if we add ecological constraints the level of harvest associated with the production in the monetary stationary state might exceed what is regenerated by the biomass stock. Therefore, the system still collapses, if the ratio of c_m and r is between the two stability frontiers. If the stationary state level of harvest is equal to what is regenerated by the ecology the system reaches an ecologically and monetarily stable stationary state (see left graph of Figure 3). Consequently, if we assume that consumption out of wealth is exogenously given and below 0.44, which is the value for which instability may occur, interest rates must be low for the overall economy to be within ecological boundaries and work at a sustainable level of production.

6 Conclusion

Using a simple baseline model, this paper examines the interactions of financial assets, real physical goods and services, and the physical environment. It shows that post-Keynesian stock-flow consistent models can successfully be integrated with the study of physical stocks, flow and funds as suggested by Georgescu-Roegen. The model consistently integrates a demand driven monetary economy with ecological constraints. Ecological scale introduces a non-linearity into the model, leading to rich dynamics. Depending on parameters such as interest rates or government expenditures, we can observe three different states of the model: The first state is a stable, non-growing economy compatible with ecological stability. In the second case, the economy approaches a monetary stationary state while constantly degrading its ecological environment, causing an ecological and economic collapse. In the third case, exponential accumulation of monetary assets through interest income leads to ever increasing demand, ecological degradation and finally breakdown. One major factor driving these differences is the parameter ratio of consumption out of wealth and interest rate. If consumption out of wealth is low, low interest rates are needed for the model to converge towards a sustainable stationary state of production. The (relative) simplicity of the model makes it tractable and easier to analyze, while obviously neglecting several important aspects such as multiple goods, pricing, fixed capital, income distribution, complex financial assets or portfolio choice. Restricting our depiction to energy, even a treatment of physical mass, waste or carbon emissions is missing as provided by Taylor *et al.* (2016) or Dafermos *et al.* (2017). The integration of these concepts can profit from a growing literature in post-Keynesian and ecological economics. Combining both approaches and integrating monetary and ecological issues may be helpful to determine the conditions under which a sustainable economy is possible, a problem that is neither purely economic, nor purely environmental, nor purely physical, but rather are all of the above.

7 References

Argyris, J.H., Faust, G. and Haase, M. (2015), *An Exploration of Dynamical Systems and Chaos: Completely Revised and Enlarged Second Edition*, Springer Berlin Heidelberg, Berlin Heidelberg.

Berg, M., Hartley, B. and Richters, O. (2015), "A stock-flow consistent input–output model with applications to energy price shocks, interest rates, and heat emissions", *New Journal of Physics*, Vol. 17 No. 1, p. 15011.

Bezemer, D.J. (2010), "Understanding financial crisis through accounting models", *Accounting, Organizations and Society*, Vol. 35 No. 7, pp. 676–688.

Buttlar, H.v. (1975), "Umweltprobleme", *Physikalische Blätter*, Vol. 31 No. 4, pp. 145–155.

Dafermos, Y., Nikolaidi, M. and Galanis, G. (2017), "A stock-flow-fund ecological macroeconomic model", *Ecological Economics*, Vol. 131, pp. 191–207.

Dale, M., Krumdieck, S. and Bodger, P.S. (2012), "Global energy modelling — A biophysical approach (GEMBA) Part 1: An overview of biophysical economics", *Ecological Economics*, Vol. 73, pp. 152–157.

Daly, H.E. (1985), "The Circular Flow of Exchange Value and the Linear Throughput of Matter-Energy: A Case of Misplaced Concreteness", *Review of Social Economy*, Vol. 43 No. 3, pp. 279–297.

Daly, H.E. (1992), "Allocation, distribution, and scale: towards an economics that is efficient, just, and sustainable", *Ecological Economics*, Vol. 6 No. 3, pp. 185–193.

Dobusch, L. and Kapeller, J. (2012), "Heterodox united vs. mainstream city? Sketching a framework for interested pluralism in economics", *Journal of Economic Issues*, Vol. 46 No. 4, pp. 1035–1058.

Galbraith, J.K. (2009), "Amen", *Thought & Action*, pp. 85–97.

Georgescu-Roegen, N. (1971), *The Entropy Law and the Economic Process*, Harvard University Press, Cambridge, Mass.

Godley, W. and Lavoie, M. (2012), Monetary economics: an integrated approach to credit, money, income, production and wealth, 2nd ed., Palgrave Macmillan, Basingstoke, New York.

Gowdy, J.M. (1991), "Bioeconomics and post Keynesian economics: a search for common ground", *Ecological Economics*, Vol. 3 No. 1, pp. 77–87.

Heyes, A. (2000), "A proposal for the greening of textbook macro: 'IS-LM-EE'", *Ecological Economics*, Vol. 32 No. 1, pp. 1–7.

Holt, R.P.F., Pressman, S. and Spash, C. (Eds.) (2009), *Post Keynesian and Ecological Economics: Confronting Environmental Issues*, Edward Elgar, Cheltenham, U.K.

Keynes, J.M. (1936), The general theory of employment, interest and money, Harcourt, Brace, New York.

Koo, R. (2011), "The world in balance sheet recession: causes, cure, and politics", *Real-world economics review*, Vol. 58 No. 12, pp. 19–37.

Kronenberg, T. (2010), "Finding common ground between ecological economics and post-Keynesian economics", *Ecological Economics*, Vol. 69 No. 7, pp. 1488–1494.

Kümmel, R. (2011), *The Second Law of Economics: Energy, Entropy, and the Origins of Wealth*, Springer, New York, Dordrecht, Heidelberg, London.

Lavoie, M. (2006), "Do Heterodox Theories Have Anything in Common? A Post-Keynesian Point of View", *European Journal of Economics and Economic Policies: Intervention*, Vol. 3 No. 1, pp. 87–112.

Lawn, P.A. (2003), "On Heyes' IS–LM–EE proposal to establish an environmental macroeconomics", *Environment and Development Economics*, Vol. 8 No. 01, pp. 31–56.

Ott, K. and Döring, R. (2008), Theorie und Praxis starker Nachhaltigkeit, Beiträge zur Theorie und Praxis starker Nachhaltigkeit, Bd. 1, 2nd ed., Metropolis, Marburg.

Perman, R. (2011), *Natural resource and environmental economics,* 4. ed., Pearson Education Limited, Harlow, Munich.

Rezai, A. and Stagl, S. (2016), "Ecological macroeconomics: Introduction and review", *Ecological Economics*, Vol. 121, pp. 181–185.

Richters, O. and Siemoneit, A. (2017), "Consistency and stability analysis of models of a monetary growth imperative", *Ecological Economics*, Vol. 136, pp. 114–125.

Rockström, J., Steffen, W., Noone, K., Persson, Å., Chapin, F.S., Lambin, E.F., Lenton, T.M., Scheffer, M., Folke, C., Schellnhuber, H.J., Nykvist, B., Wit, C.A. de, Hughes, T., van der Leeuw, S., Rodhe, H., Sörlin, S., Snyder, P.K., Costanza, R., Svedin, U., Falkenmark, M., Karlberg, L., Corell, R.W., Fabry, V.J., Hansen, J., Walker, B., Liverman, D., Richardson, K., Crutzen, P. and Foley, J.A. (2009), "A safe operating space for humanity", *Nature*, Vol. 461 No. 7263, pp. 472–475.

Stocker, T.F., Qin, D., Plattner, G.K., Tignor, M., Allen, S.K., Boschung, J., Nauels, A., Xia, Y., Bex, V. and Midgley, P.M. (Eds.) (2013), *Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*, Cambridge University Press, Cambridge, U.K.

Taylor, L., Rezai, A. and Foley, D.K. (2016), "An integrated approach to climate change, income distribution, employment, and economic growth", *Ecological Economics*, Vol. 121, pp. 196–205.

Wainwright, J. and Mulligan, M. (2013), *Environmental modelling: Finding simplicity in complexity,* 2nd edition, Wiley-Blackwell, Chichester.

WCED (1987), *Our common future*, Oxford University Press, Oxford, New York.

Appendix

A. Derivation of Stability conditions

We have the following system of equations:

$$Y_t^p = \frac{C_t^T}{p + \gamma_\epsilon C_t^T / S_{t-1}} \Leftrightarrow Y_t^p = \left(\frac{p}{C_t^T} + \frac{\gamma_\epsilon}{S_{t-1}}\right)^{-1}, \tag{25}$$

$$S_t = S_{t-1} + S_{t-1}\, a\left(1 - \frac{S_{t-1}}{S_{max}}\right) - \gamma_\epsilon Y_t^p \; with\; 0 < a < 2.6, \tag{26}$$

$$M_t = M_{t-1} + (1-\theta)(G + rM_{t-1}) - \theta Y_t^p\, p, \tag{27}$$

with

$$C_t^T = c_y(1-\theta)\left(p\, Y_{t-1}^p + G + rM_{t-1}\right) + c_M M_{t-1}, \tag{28}$$

$$Y_{t,\epsilon}^p = Y_t^p(1+\epsilon).$$

With $M_t = M_{t-1} = M^*$ we can derive from (27)

$$M^* = \frac{\theta Y^{p*} p - (1-\theta)G}{(1-\theta)r}. \tag{29}$$

Accordingly, we can derive from (26)

$$Y^{p*} = \frac{a}{\gamma} S^*\left(1 - \frac{S^*}{S_{max}}\right) \Leftrightarrow \frac{\gamma_\epsilon}{S^*} = \frac{a}{Y^{p*}}\left(1 - \frac{S^*}{S_{max}}\right). \tag{30}$$

From (25) we can derive

$$\frac{1}{Y^{p*}} = \frac{p}{C^T} + \frac{\gamma_\epsilon}{S^*}. \tag{31}$$

Substitute (29) in (28) to obtain

$$C^T = Y^{p*} p\left(c_y + \frac{c_m\, \theta}{(1-\theta)r}\right) - \frac{c_m}{r} G. \tag{32}$$

Substitute (30) and (32) in (31) and extract Y^* to obtain

$$Y^{p*}\left(\frac{1}{1 - a\left(1 - \frac{S^*}{S_{max}}\right)} - c_y - \frac{c_m\, \theta}{(1-\theta)r}\right) = -\frac{c_m}{r\, p} G, \tag{33}$$

$$\Leftrightarrow \frac{a}{\gamma_\epsilon} S^*\left(1 - \frac{S^*}{S_{max}}\right)\left(\frac{1}{1 - a\left(1 - \frac{S^*}{S_{max}}\right)} - c_y - \frac{c_m\, \theta}{(1-\theta)r}\right) = -\frac{c_m}{r\, p} G.$$

B. Derivation of the global instability

The model is unstable for $F(S^*) = \frac{1}{1-a\left(1-\frac{S^*}{S_{max}}\right)} - c_y - \frac{c_m\, \theta}{(1-\theta)r} > 0$ if $S^* \in [0, S_{max}]$.

With $c_r = \frac{c_m}{r}$ we can derive that this condition is fulfiled for $c_r < \left(\frac{1}{1-a\left(1-\frac{S^*}{S_{max}}\right)} - c_y\right)\frac{1-\theta}{\theta}$.

Derive $\min\left(\left(\frac{1}{1-a\left(1-\frac{S^*}{S_{max}}\right)} - c_y\right)\frac{1-\theta}{\theta}\right)$ if $S^* \in [0, S_{max}]$ to obtain the minimum possible value of $\left(\frac{1}{1-a\left(1-\frac{S^*}{S_{max}}\right)} - c_y\right)\frac{1-\theta}{\theta}$. This is the case for $S^* = S_{max}$. After substitution we can state

$$\text{if } c_r < \frac{(1-c_y)(1-\theta)}{\theta} \Rightarrow F(S^*) > 0 \ \forall S \in [0, S_{max}] \tag{34}$$

C. Derivation of the Jacobian

Deriving the partial derivatives of each variable of the system of equations given by Equations (25) to (27) and using the following condition for the stationary state value, we can derive the Jacobian matrix. For Y_t we know[8]

$$Y_t = \left(\frac{p}{C_t^T(Y_{t-1}, M_{t-1})} + \frac{\gamma_\epsilon}{S_{t-1}}\right)^{-1}. \tag{35}$$

At the fixed point we know from Equations (30), (31) and (32):

$$\left(C^{T*}\right)^{-1} = \left[c_y\,(1-\theta)(p\,Y^{p*} + G) + \left(c_m + c_y(1-\theta)r\right)M^*\right]^{-1} = \frac{1}{Y^{p*}p}\left(1 - a + a\frac{S^*}{S_{max}}\right), \tag{36}$$

$$\frac{Y^{p*}}{S^*} = \frac{a}{\gamma_\epsilon}\left(1 - \frac{S^*}{S_{max}}\right). \tag{37}$$

Using these relations, we can simplify the partial derivatives at the fixed point dependent on S^*:

$$\frac{dY_t}{dY_{t-1}} = (Y^{p*})^2 p^2 c_y (1-\theta)(C^T)^{-2} = c_y(1-\theta)\left(1 - a + a\frac{S^*}{S_{max}}\right)^2, \tag{38}$$

$$\frac{dY_t}{dM_{t-1}} = (Y^{p*})^2 p\left(c_m + c_y\,(1-\theta)r\right)(C^T)^{-2} = \frac{1}{p}\left(c_m + c_y\,(1-\theta)r\right)\left(1 - a + a\frac{S^*}{S_{max}}\right)^2, \tag{39}$$

$$\frac{dY_t}{dS_{t-1}} = \gamma_\epsilon\left(\frac{Y^{p*}}{S^*}\right)^2 = \frac{a^2}{\gamma_\epsilon}\left(1 - \frac{S^*}{S_{max}}\right)^2, \tag{40}$$

$$\frac{dM_t}{dY_{t-1}} = -\theta p c_y(1-\theta)\left(1 - a + a\frac{S^*}{S_{max}}\right)^2, \tag{41}$$

$$\frac{dM_t}{dM_{t-1}} = (1 + (1-\theta)r) - \theta\left(c_m + c_y\,(1-\theta)r\right)\left(1 - a + a\frac{S^*}{S_{max}}\right)^2, \tag{42}$$

$$\frac{dM_t}{dS_{t-1}} = -\theta p\gamma_\epsilon\left(\frac{Y^{p*}}{S^*}\right)^2 = -\theta p\frac{a^2}{\gamma_\epsilon}\left(1 - \frac{S^*}{S_{max}}\right)^2, \tag{43}$$

$$\frac{dS_t}{dY_{t-1}} = -\gamma_\epsilon c_y(1-\theta)\left(1 - a + a\frac{S^*}{S_{max}}\right)^2, \tag{44}$$

$$\frac{dS_t}{dM_{t-1}} = -\frac{\gamma_\epsilon}{p}\left(c_m + c_y\,(1-\theta)r\right)\left(1 - a + a\frac{S^*}{S_{max}}\right)^2, \tag{45}$$

$$\frac{dS_t}{dS_{t-1}} = 1 + a - 2a\frac{S^*}{S_{max}} - \gamma_\epsilon^2\left(\frac{Y^{p*}}{S^*}\right)^2 = 1 + a - 2a\frac{S^*}{S_{max}} - a^2\left(1 - \frac{S^*}{S_{max}}\right)^2. \tag{46}$$

With $Z_A = (1 - a + a\,S^*/S_{max})^2$ and $Z_B = \left(1 - \frac{S^*}{S_{max}}\right)^2$ this yields the Jacobian

8 All variables used here are in Joule per time step, except of M with is in € per time step. Therefore, we leave aside p in the upper index of the variables to simplify the variable names.

$$J = \begin{pmatrix} c_y(1-\theta)\, Z_A & \frac{1}{p}(c_v + c_m(1-\theta)r)\, Z_A & \frac{1}{\gamma} a^2 Z_B \\ -\theta p c_y(1-\theta)\, Z_A & (1+(1-\theta)r) - \theta(c_v + c_m(1-\theta)r)\, Z_A & -\frac{\theta p}{\gamma} a^2 Z_B \\ -\gamma c_y(1-\theta)\, Z_A & -\frac{\gamma}{p}\left(c_v + c_y(1-\theta)r\right) Z_A & 1 + a - 2a\frac{S^*}{S_{max}} - aZ_B \end{pmatrix}. \tag{47}$$

D. Derivation of $c_r = H(S^*)$

We can convert (33)) to

$$H(S^*) \coloneqq c_r = \frac{a\, S^*(S_{max} - S^*)\left((S_{max} - S^*)a\, c_y + S_{max}\left(1 - c_y\right)\right)\, (1-\theta)p}{\left((a\theta p S^*(S_{max} - S^*) + G\gamma(1-\theta)S_{max})\right)\left(S_{max} - a(S_{max} - S^*)\right)}. \tag{48}$$