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University of Oldenburg

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Integrating Energy Use into Macroeconomic Stock-Flow Consistent Models

Presented by
Oliver Richters

First Supervisor: Prof. Dr. Ulrike Feudel
Second Supervisor: Prof. Dr. Hans-Michael Trautwein

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für Imke.

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Abstract

This thesis conceptualizes a method to analyze the structural dependencies between the physics of energy use and the economy. By synthesizing stock-flow consistent models, input–output models, and aspects of ecological macroeconomics, a discrete dynamical model is developed to simultaneously study monetary flows through the financial system, flows of produced goods and services through the real economy, and flows of physical materials through the natural environment. First, the stability properties of the model are analyzed using bifurcation theory, delineating a generalized, multi-sectoral version of the Sraffian maximum rate of profit, and instabilities induced by inventory oscillations. Further analysis challenges claims that 0% interest rates are a necessary condition for a stationary economy. Second, one particular application of the model is illustrated by applying it to energy related problems such as rebound effects, and to assessing the contribution of energy price shocks to recessions caused by changes in Energy Returned on Energy Invested or price markup. Third, a minimal single-layer atmosphere climate model is used to demonstrate that the effect of anthropogenic heat flux from energy conversion on climate change should be taken into account in climate modeling once long-term growth scenarios are examined. In further research the model can contribute to an integrated assessment of pressing multidimensional problems such as climate change and the transformation to a sustainable economy, which equally relate to the economic, environmental and ecological sphere.

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Acronyms

ABM Agent-Based Model

DIO Dynamic Input–Output (model)

EIO Environmental Input–Output (model)

EROI Energy Returned on Energy Invested

FF Flow of Funds

GDP Gross Domestic Product

IO Input–Output (model)

SAM Social Accounting Matrix

SFC Stock-Flow Consistent (model)

1 Introduction

Energy use is one of the key aspects of economic activity, and directly linked to the standard of living (Kander et al., 2013; Kümmel, 2011). At the same time, most of the environmental threats identified within the debate about 'planetary boundaries' (Rockström et al., 2009; Steffen et al., 2015) are strongly linked to the use of engineered systems and the growth of energy and resource consumption (Krumdieck, 2013). Thus navigating the transformation towards a sustainable economy respecting ecological limits is of the key issues faced by modern society. This includes the implementation of sustainable energy systems, which are technically reasonable as well as economically acceptable (Kreith and Krumdieck, 2013, p. 1). Technological changes within the energy sector are necessary, but it is an open question whether this technological progress is sufficient for decreasing environmental impact: Improved efficiency or new technologies may facilitate economic growth and lead to an increase in consumption and ecological impact (Madlener and Alcott, 2009). Therefore, there is a risk of focusing too much on purely technological solutions such as improvements of energy efficiency and the development of renewable energy sources (Jackson, 2009). Addressing the question of a sustainable economy requires the integration of energy use and emissions into macroeconomic models.

In most of economic theory the role of energy and natural resources is underemphasized or neglected. However, ecological economists such as R. U. Ayres, L. W. Ayres, et al. (2003), R. U. Ayres and Warr (2005) or Kümmel (2011) have treated energy as a discrete factor of production. Similar to most economic models in the tradition of general equilibrium models, they abstract from institutional details of money creation and monetary flows which play a central role in real-world macroeconomic dynamics (Godley and Lavoie, 2007; Graziani, 2003). While the significance of these abstractions has been discussed intensively, there is certainly a lack of macroeconomic models with explicit treatment of both, a monetary economy and aspects of ecological economics (Gowdy, 1991; Kronenberg, 2010a).

The scope of this thesis is to contribute to investigating the interrelations between the monetary economy and the physical environment by synthesizing elements from post-Keynesian stock-flow consistent (SFC) models, input–output (IO) analysis, and physics (thermodynamics in particular). A conceptual dynamical economic model in discrete time is presented, analyzed and applied to energy price shocks, and rebound effects, while human heat emission are studied in a simple climate model.

The next chapter gives an introduction to these fields and highlights common ground in section 2.5, underlining that there is no serious theoretical impediment to the integration of the approaches. Chapter 3 presents a conceptual model combining aspects of both SFC and IO approaches. A stability analysis is performed in chapter 4, studying the maximum rate of profit and inventory oscillations, and contributing to the discussion of whether a stationary economy is compatible with positive interest rates. In chapter 5, rebound effects and the impact of energy price shocks caused by higher markups or declining Energy Returned on Energy Invested (EROI) are studied. Chapter 6 considers heat emissions from energy conversion in light of a minimal single-layer atmosphere climate model showing the impact of anthropogenic heat flux on climate change. Section 7 discusses the analytical framework, suggesting several model extensions from different schools of economic thinking. The conclusion assesses the relevance of the contribution.

Some theoretical considerations, the model structure, the section on heat emissions and preliminary results of the stability analysis have been published together with Matthew Berg (University of Missouri Kansas City) and Brian Hartley (The New School for Social Research) in New Journal of Physics **17** 015011, January 2015, and as a conference paper (Berg et al., 2015a,b).

2 Framework and Methods

2.1 The Macroeconomic Significance of Energy

Several studies underline that the wealth of industrial nations has grown thanks to energy services over the last several centuries and in particular since the beginning of the industrial revolution (Kander et al., 2013; Kümmel, 2011; Wrigley, 2010). Nevertheless, the significance of resources and energy has generally been downplayed in most modern theories (Binswanger and Ledergerber, 1974; Kümmel, 2011). In contrast, some pre-classical, physiocratic, and early nineteenth century classical economists were aware of the physical side of economic activity, similar to the contemporary *ecological economists* (Christensen, 1989; Cleveland, 1987; Røpke, 2004). The latter object to most economic models because of their focus on the circular flow of exchange value (i.e. money), rather than on the physical throughput of natural resources from which all goods and services are ultimately derived (R. U. Ayres, 1978; Cleveland, 1999; Daly, 1985; Frondel and Schmidt, 2004; Georgescu-Roegen, 1971). Most economists interpret energy services as enhanced labor or capital productivity associated with technological progress (Kümmel, 2011, p. 52), which is considered to be the biggest contribution to economic growth (Solow, 1956; Blanchard and Illing, 2014, pp. 321ff). But ascribing growth to this 'amorphous force that can increase productive power without limit' (Gowdy et al., 2009, p. 206) has been critized for leaving the 'main factor in economic growth unexplained' (Solow, 1994).

Some economists have considered energy E as a factor of production, sometimes in combination with materials M, but have underestimated the importance of these factors. The responsiveness of output to a marginal change of one production factor in the neoclassical approach is given by its output elasticity $E_{y,x}$, the point elasticity of output y of an entity with respect to a production factor x (Kümmel and Lindenberger, 2014):

$$E_{y,x} = \frac{x}{y}\frac{\partial y}{\partial x}. \tag{1}$$

The theory assumes that in equilibrium, this should be identical to the cost share of the production factor. Energy costs represent about five percent of production costs; consequently, the output elasticity of energy has been estimated to be 0.05. As this is low compared to labor with 0.7 or capital with 0.25 during recent decades in OECD countries, energy has been left out of most economic models (Gowdy et al., 2009, p. 207; Manne, 1978), see Kümmel (2011, pp. 180–212) for a longer discussion. Cost share and output elasticity are not necessarily equal once a third factor is added that is not independent of the other two (R. U. Ayres and Warr, 2005, p. 16). This is the case here, since 'capital in the absence of energy is functionally inert', and technical engineering constraints limit substitution (Kümmel, 2011, p. 195). Based on a general equilibrium framework extended by incorporating energy as a production factor, Kümmel uses non-linear optimization with generalized shadow prices on real data to calculate time-averaged output elasticities of 0.37 for capital, 0.11 for routine labor, and 0.52 for energy, while the remaining residual of 0.12 is ascribed to the residuum called creativity (ibid., pp. 180, 212). Similar values are found by R. U. Ayres, L. W. Ayres, et al. (2003) and R. U. Ayres and Warr (2005). Using these elasticities, energy accounts for most of the growth attributed to technological progress (Kümmel, 2011, p. 221). This indicates that postulating an identity between factor costs and output elasticities is flawed, and the neglect of energy is without solid foundation.

The significance of these findings is underlined by the International Monetary Fund, which investigates the impact of lower oil supply in its World Economic Outlook, stating that 'if the contribution of oil to output proved much larger than its cost share, the effects could be dramatic, suggesting a need for urgent policy action' (International Monetary Fund, 2011, p. 109). Given the naturally constrained supply of fossil fuels, the connection between energy and the economy must be understood in order to avert potential challenges to the modern global industrial system, which currently depends categorically on fossil fuels and other non-renewable energy sources (Heinberg, 2007). A declining capacity to extract energy has sometimes been an important trigger of societal collapse (Homer-Dixon, 2006, p. 36; Tainter, 1988, pp. 91–122). This not only has historical implications, but could also potentially impact theoretical accounts of modern business cycles, as every US recession since World War II was accompanied by rising energy prices (Hamilton, 1983, 2013; Murphy and Hall, 2011). In section 5.1, we suggest that this could have been caused by a decline in effective demand due to higher energy prices. Other studies have underlined the contemporary significance of energy in terms of the 'Energy Returned on Energy Invested' (EROI), which is the usable energy acquired divided by the amount of energy expended to extract and process that energy resource (Cleveland et al., 1984). It is an open question whether unconventional oil fields allow for an extraction velocity comparable to conventional fields, and at lower EROI, economic growth will become 'harder to achieve and come at an increasingly higher financial, energetic and environmental cost' (Murphy, 2014). The impact of declining EROI is studied in section 5.2.

In order for economic activity to be environmentally sustainable, such that it 'meets the needs of the present without compromising the ability of future generations to meet their own needs' (World Commission on Environment and Development, 1987), it must be the case that the ecosystem can absorb waste and recycle the inputs which are required for physical production (Daly, 1992, p. 186). Therefore, the physical and environmental sustainability of the economy can best be analyzed by considering the economy as an open subsystem of the larger but finite physical ecosystem, as energy usage entails heat and particle emissions (Kümmel, 2011). While energy conservation may provide a partial solution to this problem, there are inescapable thermodynamic limits to energy efficiency which may limit decoupling of resource use and economic growth. Furthermore, energy use may not necessarily decline even if energy conservation measures render such a decline technically feasible because of rebound effects (Kümmel, 2011; Madlener et al., 2009), studied and described in section 5.3. In a sustainable economy, energy available for production will not be limited by the availability of energy *as such*, but rather by the *capacity to extract renewable resources* due to the fact that the buildup of the capital stock requires energy input (Dale et al., 2012a,b). For these reasons, some ecological economists argue that the necessity of adapting to planetary boundaries and resource extraction limits may decrease energy supply, and the constraint of this main driver of economic growth may render a stationary economy or economic degrowth unavoidable (Jackson, 2009; Kallis et al., 2012; Pueyo, 2014).

2.2 Stock-Flow Consistent (SFC) Models

The post-Keynesian approach underlines the significance of a *monetary economy* (Godley and Lavoie, 2007; Graziani, 2003) and objects the neutrality of money used in most neoclassical general equilibrium models (Blanchard et al., 2014). Gerard Debreu, one of the inventors of the Arrow-Debreu general equilibrium model was well aware that his approach does not address the 'important and difficult question' of the 'integration of money in the theory of value' (Debreu, 1959, p. 36), and Graziani (2003, p. 15) argues that it 'seems impossible to reconcile a rigorously defined equilibrium position with the presence of money'. General equilibrium theory relies on the concept of a real economy where money is only incorporated as a mean of exchange in order to improve efficiency of barter: "And as production is instantaneous, while supply is brought into equivalence with demand through the market-clearing process, there is no systemic need and therefore no essential place for loans, credit money or banks. The concept of 'money' is indispensable, yet money is an asset to which there is not, in general, a counterpart liability and which often has no accounting relationship to other variables" (Godley and Lavoie, 2007, p. 4).

Most general equilibrium models assume that money does not differ fundamentally from other goods and that the stock of money is exogenously provided by the central bank, and banks act as financial intermediaries, lending deposited funds. Similarly, many models in econophysics assume that the quantity of money as an exogenously given stock is conserved, and the focus is placed squarely upon the *exchange* of wealth (Chakraborti and Chakrabarti, 2000; Chatterjee et al., 2005; Patriarca et al., 2010), ignoring both the role of production and credit creation in economic activity and disregarding standard definitions of economic concepts such as transactions and income (Chen et al., 2014; Gallegati et al., 2006; Schmitt et al., 2014).

Both approaches are inconsistent with a substantial body of work in post-Keynesian monetary theory, along with the statements of central bankers who say that credit money is created endogenously via loan origination, and the central bank reacts on demand for reserves by banks (Holmes, 1969; Kumhof and Jakab, 2015; McLeay et al., 2014). The usual description of money found in most economic textbooks is therefore misleading (S. Carpenter and Demiralp, 2012; McLeay et al., 2014). Banks do not lend reserves, but rather create credit *ex nihilo* by simultaneously expanding both sides of their balance sheets, creating an asset of the bank (a loan) and a liability of the bank (a deposit) (Graziani, 2003; Moore, 1988). Schmitt et al. (2014) depict the process of money creation with a limited analogy to physics, in which asset units are considered as money and liability units as 'antimoney', allowing the expansion and contraction of the money stock to occur endogenously. The implication is that investment does not require precedent saving as assumed by neoclassical economists, but that investment and credit creation can lead to savings (Graziani, 1988, p. 283).

One effort to explicitly represent the dynamics of debt, finance, and other monetary factors has been the post-Keynesian stock-flow consistent (SFC) approach. SFC models are a class of structural macroeconomic models grounded by a detailed and careful articulation of accounting relationships. Though post-Keynesian authors criticized the aggregation procedures of neoclassical authors, most SFC models are formulated as sectoral models, similar to a mean field approach, but the structure of the models allows for disaggregation. The passage of time is usually defined in discrete periods of equal finite length (Taylor, 2008; Tobin, 1982). The basis of this method can be found in the work of Copeland and Stone, who advocated for macroeconomic models developed with the use of social accounting matrices (SAMs) that tabulate stocks and flows of funds within the national accounts (Copeland, 1949), and is also similar to the 'Saldenmechanik' ('balance sheet mechanics') by Stützel (1978). Godley, Lavoie, and a number of other authors expanded this approach into a family of applied macroeconomic models that respect accounting identities and are closed with behavioral assumptions based on post-Keynesian theory (Godley and Lavoie, 2007; Lavoie and Godley, 2001; Zezza and Dos Santos, 2004). Perhaps the single most important advantage of the SFC approach is that it enables the modeler to easily create scalable representations of institutional structures with an explicit monetary dimension. The central importance of attention to financial detail was illustrated by the failure of the macroeconomics profession to anticipate the 2007–2008 Global Financial Crisis, which was predicted nearly exclusively by those who deployed implicit or explicit macro-accounting frameworks (Bezemer, 2010; Galbraith, 2009; Koo, 2011).

The accounting relationships are displayed in balance sheets or T-accounts. Assets are shown on the left side of the T-account, while liabilities and net worth are shown on the right side of the T-account, as visible for each sector in figure 1 on page 24. In accounting, a fundamental equation known as the *balance sheet equation* states that:

$$\text{Assets} = \text{Liabilities} + \text{Net Worth}. \tag{2}$$

This means that the left side of the T-account is by definition always equal to the right side of the T-account, and they must always expand or contract equally. This double entry bookkeeping is a symmetry principle, and is why balance sheets are called 'balance' sheets. Accounting identities remove degrees of freedom from potential macroeconomic outcomes, forming a skeletal structure that can be closed by a number of competing theoretical arguments.

SFC models are constructed by tabulating the balance sheets and transactions of the different sectors. Each row of the transaction flow matrix represents a transaction, while each column represents a different sector. The transaction flow matrix used in the model is displayed in table 2. In a manner consistent with flow of funds (FF) accounts, sources and uses of funds are represented, respectively, by positive and negative signs. All rows and columns sum to zero for financial transactions, since every financial asset has a corresponding financial liability. Because all flows necessarily accumulate to stocks, the flows of funds represented on the transaction flow matrix directly imply stock and balance sheet adjustments, which creates additional constraints. This accounting verification process is designed to ensure the model's internal logical consistency by removing potential 'black holes' (Godley, 1996, p. 7), and by respecting a 'fundamental law of macroeconomics analogous to the principle of conservation of energy in physics' (Godley and Cripps, 1983, p. 18).

In the Cambridge Capital Controversies (Harcourt, 1972), post-Keynesian authors discovered logical problems with the aggregate neoclassical production function, such as reswitching and reverse capital-deepening. Post-Keynesians therefore rejected the neoclassical aggregate capital stock and the neoclassical production function (Felipe and McCombie, 2006), and do not posit an identity between output elasticity and cost share (Christensen, 1989, p. 29). The neoclassical school largely deflected these theoretical critiques by asserting that the properties of production functions had empirical validity, even if the aggregate production function was logically inconsistent. However, an extensive literature demonstrates that many regressions upon deflated monetary data simply measure distributional variables in the national accounts and do not convey meaningful information about parameter values of the production function or technological relationships (Felipe et al., 2006; Shaikh, 1974). This is to say that the regressions are simply estimating accounting identities, which are true by definition but concern a different question. In contrast, post-Keynesian authors view production as a discrete and sequential technically determined process with limited possibility for immediate substitution, similar to input–output (IO) models.

2.3 Input–Output (IO) Models

Input–output models provide a detailed treatment of production and of the flow of real goods and services through the economy, and are commonly applied to analyze interactions and feedback effects between mutually interdependent industrial sectors. The IO approach can be traced back to classical authors (Kurz and Salvadori, 2000). The first modern IO model was created by Alfred Kähler (1933), and developed by Wassily Leontief (1976) into the sort of large scale empirical model now routinely produced by statistical agencies in countries across the globe.

IO tables provide a static snapshot view of the economy, assuming constant returns to scale. They are displayed in matrix notation ('Leontief matrix'), where each column represents inputs to a specific sector, while each row shows the output from a given sector to the rest of the economy. For an economy with n sectors, a $n \times n$ matrix $\mathbf{a}$ is used, where $a_{ij} \geq 0$ is a flow of inputs produced by sector i to sector j in order to produce one unit of output j. To produce the gross outputs of the different sectors, which are displayed as the elements of a vector $\boldsymbol{x}$, a different vector $\mathbf{a}\boldsymbol{x}$ is required as intermediate inputs. Therefore, in every time period, gross output and final demand $\boldsymbol{d}$ (also referred to as net output or GDP, gross domestic product) are coupled by:

$$\boldsymbol{x} = \mathbf{a}\boldsymbol{x} + \boldsymbol{d}, \tag{3}$$

$$\boldsymbol{x} = (\mathbf{1} - \mathbf{a})^{-1} \boldsymbol{d}. \tag{4}$$

To obtain a unique and positive solution, $(\mathbf{1} - \mathbf{a})$ has to be invertible and the principal minors have to be positive, known as the Hawkins-Simon conditions (Hawkins and Simon, 1949; Miller and Blair, 2009, pp. 58ff.), in order to guarantee that each subsystem is 'productive' such that it requires less input than it produces in terms of output.

In general, environmental impacts can be deduced from a combined environmental and economic accounting (United Nations et al., 2005). To track the flow of income and its distribution among sectors, SAMs have been attached to IO models (Miller et al., 2009, pp. 499–542), and IO models which incorporate SAMs are not dissimilar in general form and purpose to the model as presented later. Note that SAMs, which served as a basis for the development of SFC models, are a linkage between IO and SFC models. However, effective demand is often left exogenous in these models and is not determined by a logically consistent system of SFC FF equations, and the role of monetary dynamics has been left relatively unexplored in IO models (Caiani et al., 2014). As mentioned, SFC models focus primarily on explicating flows of financial funds, and therefore often underemphasize real production. Indeed, most SFC models only include a single productive sector, and nearly all multi-sectoral models abstract from intermediate production. Aspects of the IO approach, which provides a detailed picture of a complex multi-sectoral economy, can therefore be used to import a more refined analysis of the real economy into a SFC framework.

2.4 Dynamical System Theory

The mathematical models introduced in this thesis are studied with methods from dynamical system theory. In general, a dynamical system is one whose state changes with time according to mathematical rules. This evolution of a dynamical system is given by the iterated application of a transition function, and the state of the system depends on its history. If, as usual, these rules are not changing with time, the system is called *autonomous*. Dynamical system theory tries to understand the ‘essential and dominant features’ of a complicated system, in particular, the dependence of the qualitative behavior of a system on certain parameters of the transition rule and on initial conditions (Wiggins, 2003, p. 2). The focus of this thesis lies on both the asymptotic behavior of the system and the analysis of instabilities, studying whether the behavior of such a system is sensitive to small perturbations of initial conditions. The setting for the study of dynamical systems involves *space*, *time* and *time evolution* (Hasselblatt and Katok, 2002, pp. vii, 4).

The *phase space* M is a complete metric space, often a manifold locally diffeomorphic to a Banach space, with the easiest example being an Euclidean space. *Time* may be discrete (time steps) or continuous (steplessly) and is therefore parametrized by $\mathbb{Z}$ or $\mathbb{R}$, sometimes restricted to non-negative numbers in the case of irreversible processes. At any time t, a dynamical system has a *state*, a vector in the state space M. Together, the states form a *trajectory*, constituting the *time evolution* of a dynamical system. Mathematically, discrete dynamics are governed by difference equations, while continuous dynamics are treated with differential equations (Abraham et al., 1997, p. 6; Hasselblatt et al., 2002, p. 5). In the following, we restrict the explanation on discrete systems.

The time evolution is defined by the iteration of a map $f : M \to M$, a continuous function and endomorphism of a state space M into itself, leading to 'points of the system [that] jump along dotted lines with a regular rhythm' (Abraham et al., 1997, pp. xi, 29). Different periodic trajectories (*orbits*) may exist, such that $f^m(x) = x$ for some m. For $m \in \mathbb{Z}^+$, f^m is the composition of f with itself m times, with f^0 being the identity. The simplest orbit is the *fixed point* or *steady state* x', defined by $f(x') = x'$, thus an orbit which is a single point (Holmgren, 1996, p. 31). A *fixed point* x' is stable if, for every neighborhood N of x', there is a neighborhood $N' \subseteq N$ of x' such that if $x \in N'$ then $f^m(x) \in N \; \forall \; m > 0$. Trajectories from points 'near to' a stable fixed point remain 'near to' it for $m \in \mathbb{Z}^+$. If a fixed point x' is stable and $\lim_{m\to\infty} f^m(x) = x' \; \forall \; x$ in some neighborhood of x', then the fixed point is said to be *asymptotically stable* and the given orbit is called *attractor*. Those who are not stable are called *unstable* or *repelling*, and the orbit is called *repellor* (Arrowsmith and Place, 1990, pp. 5ff; Abraham et al., 1997, pp. 159ff).

In principle, different classes of dynamical systems exist, depending on the magnitude J of the determinant of its Jacobian matrix of partial derivatives (Ott, 2002, pp. 10ff):

$$J \equiv |\det(J(x))| \equiv |\det(\partial M(x)/\partial x)|. \tag{5}$$

If J=1, the map is volume preserving and the system is called *conservative*. If J<1, the system is *dissipative*, while J>1 implies 'a supply of energy to the system' (Anishchenko, 2007, p. 2). The latter two types are called *non-conservative* systems.

In a dynamical system governed by a linear map f, only one fixed point x' exists. To determine its stability, one has to calculate the eigenvalues of the map: An *eigenvalue* λ of a matrix f is a root of the characteristic polynomial $p(\lambda) = \det(f - \lambda \mathbb{1})$. An eigenvector of f to the eigenvalue λ is a non-zero vector $\boldsymbol{v}$ for which $f\boldsymbol{v}=\lambda\boldsymbol{v}$. The *stable eigenspace* or *manifold* relative to the fixed point is defined as the hyperplane spanned by the eigenvectors that are associated with eigenvalues of modulus smaller than one, while the *unstable manifold* or *eigenspace* is the hyperplane spanned by the eigenvectors associated with eigenvalues of modulus bigger than one. Iterations on elements of the stable eigenspace and iterations of the inverted map on elements of the unstable eigenspace converge to the fixed point. This fixed point x' is stable if all of the eigenvalues of f have absolute values strictly less than one. If a fixed point x' is a *saddle point*, meaning that one of the eigenvalues is larger than one and one smaller in absolute value, there exist both *stable manifolds* and *unstable manifolds*. The eigenvector space corresponding to eigenvalues λ where $|\lambda|=1$ is called *center manifold* W_c. A fixed point is called *hyperbolic* if it has no eigenvalues on the unit circle and therefore has no center manifolds (Devaney, 2003; Ott, 2002; Wiggins, 2003).

In the case of a two-dimensional system, the following definitions hold for hyperbolic fixed points: If all eigenvalues lie within the unit circle 1, the fixed point is called a *sink* if both eigenvalues are real, and a *spiral sink* or *stable focus* if the eigenvalues are a pair of complex conjugate eigenvalues. If the absolute value of both of the eigenvalues is bigger than 1, the fixed point is called a *source* in the case of real eigenvalues, and a *spiral source* or *unstable focus* in the case of a pair of complex conjugated eigenvalues. If one eigenvalue is bigger and the other smaller than one, the fixed point is called a *saddle point* as above.

The *basin of attraction* of a fixed point x' in the state space is defined as the *stable set* $W^s(x')$, given by the largest open set of points $\{q \in X\colon$ with $f^n(q) \to x'$ as $n \to \infty\}$, thus all states in the set converge to x' under application of f (Devaney, 2003, p. 216). The set of points where trajectories are unbounded and go off to infinity is called the *basin of infinity* (Abraham et al., 1997, p. 45). For a stable linear system, every point in the phase space is in the basin of attraction.

As parameters of the mapping function are changed, the structure of the attractors may change insignificantly at some values, but may also undergo sudden and significant changes at certain other values. These special values are called *bifurcation points*, and the sudden changes in the portrait are called *bifurcations*. They happen at non-hyperbolic fixed points, and the system exhibits the corresponding bifurcation on the center manifolds, while the behavior off these manifolds is trivial, e. g. exponentially attractive (Abraham et al., 1997; Kuznetsov, 2004).

In the case of the linear map, bifurcations occur if at least one of the eigenvalues of the mapping matrix passes the unit circle. If this eigenvalue is real and positive, we can observe an exchange of stability: The attractor at infinite value becomes stable, while the bounded fixed point loses its stability. This is called a *transcritical bifurcation* (Wiggins, 2003, pp. 504ff). If the matrix has a pair of complex conjugated eigenvalues $\lambda_{1,2}$ and their (identical) absolute value passes the unit circle if parameters are changed ($\lambda_{1,2} = e^{\pm i\eta}$), the dynamical system undergoes a *Neimark-Sacker bifurcation*: As a parameter is varied, a stable focus on the center manifold loses its stability (Kuznetsov, 2004).

2.5 Common Ground

Post-Keynesian and ecological economists criticize different aspects of general equilibrium theory, where the aggregate behavior of a market is studied assuming that the behavior of the economy can be inferred form individual, rational decisions that are taken in isolation. Through an intertemporal optimizing procedure, a general equilibrium is determined, and alternative models have been considered 'not scientific' (Kirman, 2011, p. 12). One should point out that the use of the term 'equilibrium' in economics may be misleading to physicists, because the analysis does not look at a 'rest point' of a dynamical system, but it is a static description 'of an allocation of resources to the individual consumers and firms, from which nobody, given the constraints imposed by the system, would have any interest in deviating' (ibid., p. 7). Out-of-equilibrium dynamics in macroeconomic models based on general equilibrium are therefore only considered in linear order in a neighborhood around the equilibrium, and it cannot be determined why and how the model economy settles at a specific fixed point. Unfortunately, it has been proven that given some heterogeneity in preferences and endowment among agents, multiple equilibria exist (Debreu, 1974; Mantel, 1974; Sonnenschein, 1972), which has been 'solved' by introducing the representative agent abstracting from heterogeneity (Kirman, 2011, p. 16). The economist neglect 'how the interactions between the individuals determine the state of the economy and, in particular, whether they would produce an equilibrium' (ibid., pp. 13f).

Unfortunately, the different critiques from the ecological and monetary perspective remain largely unconnected. Keynesian macroeconomic theory places great emphasis on the determination of a level of effective demand commensurate with key economic policy goals, but the ecological implications of those economic policy goals have often been neglected. Therefore, Mearman (2005) concluded that 'post-Keynesians need to embrace the environment' in order to underline the relevance of their work. In contrast, most ecological economists abstract from the influences of the monetary side of the economy, though some analyses of the monetary dimension of sustainability have been conducted by Tokic (2012), Binswanger (2013) and Wenzlaff et al. (2014). But outside this work, some misunderstandings appear, such as a common claim that a zero interest rate is a stability condition for a stationary economy (Farley et al., 2013; Löhr, 2012). We will review this argument in section 4.2. Issues such as monetary policy and interest rates can be most fruitfully discussed within a framework of ecological macroeconomics which is cognizant of the implications of financial flows of funds for the economy (Jackson et al., 2014).

Gowdy (1991), Kronenberg (2010a), and the contributors of the book edited by Holt et al. (2009) have explicitly argued that post-Keynesian economics and ecological economics share substantial common ground, and are ripe for a synthesis. Despite the need for new analytical tools to explore this relationship, relatively little concrete work to that end has thus far been completed (Rezai et al., 2013), notable exceptions include the work of Kemp-Benedict (2013), Kronenberg (2010b), the work in progress by Dafermos and Nikolaidi (2014), and the WWWforEurope project (Jackson et al., 2014). However, some previous attempts to integrate post-Keynesian and ecological economics are not SFC. Similarities have been recognized in terms of consumption, production theory, cumulative causation (path dependency), and the irreversibility of historical time (Holt et al., 2009; Kronenberg, 2010a; Lavoie, 2006). Both post-Keynesian and ecological economists emphasize the significance of fundamental 'Knightian' uncertainty, as opposed to computable probabilistic risk (Godley and Lavoie, 2007; Knight, 1921; Kolstad, 1996; Radner, 1968), and replace the axiom of perfect rationality and optimizing agents by 'reasonable rationality': Agents 'follow norms and targets, and act in line with these and the expectations that they may hold about the future' (Godley and Lavoie, 2007, p. 16). Both schools reject neoclassical aggregate production functions, but view production as a discrete and sequential technically determined process with limited substitution – because they either ask for compatibility with the laws of nature or because of the aggregation fallacies underlined by the Capital Controversies (Kronenberg, 2010a). This allows for an integration of input–output models into post-Keynesian SFC models. Some indications on how agent-based models (ABM) may be integrated to incorporate interaction and heterogeneity explicitly will be given in chapter 7.

By combining SFC models and IO models, financial flows of funds can be integrated with flows of real goods and services. Lawrence Klein, who developed large scale macroeconomic models typified by the FRB-MIT-Penn model, has noted the natural synergies between the National Income and Product accounts, the IO accounts, and the FF accounts (Klein, 2003). The approach of combining both SFC and IO models with ecological macroeconomics affords one method to unite those accounts, as suggested by Klein, and to simultaneously model monetary flows through the financial system, flows of produced goods and services through the real economy, and flows of physical materials through the natural environment. Models of this type may provide additional tools to aid macroeconomists, ecological economists, and physicists in the task of understanding the economy and the physical environment as one united and complexly interrelated system, rather than as a colloidal agglomeration of artificially separated analytical domains. These modes of analysis are required to study pressing problems such as climate change, which are neither purely economic, nor purely environmental, nor purely physical, but rather are all of the above (Rezai et al., 2013). The following chapter presents the methodology and structure of a conceptual stock-flow consistent input–output model.

It is surprising that although SFC models are dynamical systems, very few concepts of dynamical system theory are applied, even though this approach allows to study general properties of the models such as stability or long-term development reducing the need for extensive simulations. This thesis covers a very small fraction of the concepts established by the dynamical system community, but suggests that a more extended use may be helpful for rigorous analysis of macroeconomic models. Whether economic models should use continuous or discrete time has been subject to discussions, but Tobin (1982) argued that 'either representation of time in economic dynamics is an unrealistic abstraction'. Godley, one of the fathers of the SFC approach, 'preferred to work in discrete time, responding to the way the data are presented' (Taylor, 2008). As the SFC model used is formulated in the tradition of Godley and Lavoie (2007), the following model is described in discrete time.

3 Methodology: Stock-Flow Consistent Input–Output Models

This section introduces a conceptual baseline model that could serve as a point of common ground between the SFC, IO, and ecological macroeconomics approaches. A SFC model of a closed economy is coupled with an IO model. This approach is similar to the work in progress by the project WWWforEurope, where researchers are also developing a large multi-sectoral model connected to an explicit articulation of financial flows (Jackson et al., 2014). The model developed is a dynamical system represented in discrete time $t \in \mathbb{Z}$ and includes multiple (n) industry sectors, a household sector, and a government / banking system sector. However, the household sector and the government and banking system sector are both consolidated. This keeps the exposition relatively simple and tractable, and allows focus to squarely remain on the chief aim of integrating elements of an IO treatment of production into a SFC framework and on showing how the model can be applied to ecological macroeconomics. However, in a more complicated and more realistic version of the model, both the household sector and the government / banking system sector (hereafter referred to as simply 'government sector') would be deconsolidated, and heterogeneity in sectors other than the multiple industry sectors could be explicitly modeled. However, creating fully scalable models which articulate that degree of heterogeneity would likely call for an agent-based approach, as discussed in section 7.

The model simultaneously tracks the values of all flows of goods and services through the economy in both nominal terms (measured in terms of money-values) and in real terms (measured in terms of physical units of the heterogeneous real physical output of industry i). In order to more easily identify which variables are in real terms and which are in nominal terms,

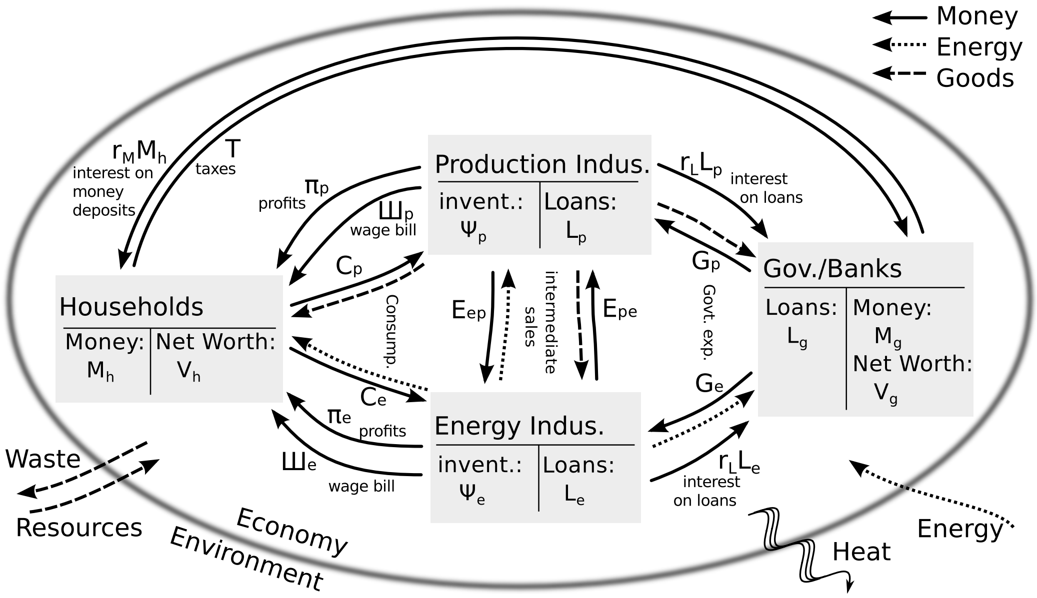

Figure 1: Stocks of sectors and flow chart of money, energy, and materials.

h: households, *g*: government / banking system sector, *p*: production sector, *e*: energy sector. For each sector, a balance sheet is shown in the form of a T-account.

Stocks: M_h: money stock of households. M_g: money issued by banks / government. $L_{p/e}$ loans of production sector / energy sector. L_g: loans made by government / banks. $\psi_{p/e}$: physical inventories of industry sectors.

Money flows: $C_{p/e}$: consumption of households. $G_{p/e}$: government expenditures. E_{ep}: money paid by production sector for energy. E_{pe}: money paid by energy sector for intermediate goods. $Ш_{p/e}$: wage bill paid by production / energy sector. $\Pi_{p/e}$: distribution of profits. T: taxes. $r_M M_h$: interest payment to households. $r_L L_{p/e}$: interest payments by production / energy sector.

Energy and **material flows:** Energy: energy extracted from the environment. Heat: heat emissions. Resources: extracted from the environment. Waste: emitted to the environment; not treated explicitly in the model, but implied.

all nominal variables are written in capital letters. The subscript $_{(t-1)}$, as in $M_{h(t-1)}$, indicates the value of the stock at the end of period $t-1$, whereas the subscript $_{(t)}$ refers to the value of the stock at the end of period t. The total number of stocks and flows in the model depends on the number of sectors n and can be calculated as $10 + 10n + 2n^2$. Therefore, the state space of the simplest model with n=2 is a 38-dimensional Euclidean space. But as many of the variables are connected closely via accounting identities or simple relations, the final number of variables in the simulations is much lower.

The simplified model with two sectors used in sections 4 and 5 is designed to facilitate an easier understanding of the core issues raised in the process of synthesizing SFC models, IO models, and ecological macroeconomics. The flow diagram in figure 1 shows the variety of financial flows and physical flows included in even a simple model with two sectors. All monetary payments (solid lines) flow from one sector to another and accumulate to the corresponding stock, providing consistency between stocks and flows.

Money flows from households to the government in the form of taxes T. Money flows from both the production sector and the energy industry to households in the form of wages $Ш_{p/e}$ and distributed profits $\Pi_{p/e}$. In turn, households spend their money on both production goods and energy goods, which creates flows of money $C_{p/e}$ back to the production and energy sectors and corresponding flows of real goods and services to the households. The government likewise buys both production goods and energy goods, which creates similar flows of both real goods and services and of money $G_{p/e}$ between the government and both of the two industries. The production industry buys energy goods as intermediate inputs, which creates flows of energy goods from the energy industry to the production industry and a corresponding flow of money E_{ep} from the production industry to the energy industry. The inverse is true for purchases of production goods as intermediate inputs by the energy industry E_{pe}. Finally, as physical raw materials are used in production, and as some raw materials are expended as waste, there are flows of physical materials between the human economy and the natural environment. Likewise, energy flows into the economy from the natural world, while heat is emitted by the economy into the natural environment. These economy-nature interactions are not *explicitly* considered in the model, but rather are simply implied. If more than two industry sectors are included, one must incorporate additional interlinkages in the diagram, but the same principles continue to apply. Differently from figure 1, which only includes two industries, the mathematical formulation of the model is done for an economy with n sectors.

The flow diagram in figure 1 also shows the balance sheets of each of the four sectors (the households sector, the government sector, the production goods sector, and the energy goods sector) in the form of T-accounts. It distinguishes two types of stocks of financial assets: money deposits and loans. Loans appear on the asset side of the government / banking system sector's balance sheet and on the liability side of industry i's balance sheet. Money deposits, on the other hand, appear on the liability side of the government / banking system sector's balance sheet, and on the asset side of the household sector's balance sheet. In the balance sheet perspective, the government sector holds assets of loans L_g on the left side of its T-Account, while it has liabilities of money deposits M_g on the right side. The difference between its assets and liabilities determines its net worth V_g, also shown on the right side of the T-account.

The money stock held by households is designated as M_h, which is always equal to M_g, because the consistent accounting in the model ensures that this will be the case, without the need for an explicit equilibrium condition equation specifying that the money 'supplied' by the government sector is equal to the money 'demanded' by the household sector.

In addition to stocks of financial assets (money deposits and loans), stocks of real assets also appear on balance sheets. A heterogeneous vector of inventories consisting of all the unsold output of each industry i at the end of each period constitutes the real assets of the model economy. These inventories are denoted by ψ_i, each held on the balance sheet of the corresponding industry sector, and are valued at unit costs. The monetary value of the stock of inventories at unit costs is symbolized by Ψ_i. The production goods industry holds assets of production good inventories Ψ_p on the left, counterbalanced by loans L_p on the right. The energy goods industry similarly holds assets of energy good inventories Ψ_e on the left, counterbalanced by loans L_e on the right. It is assumed as a simplification that industries do not hold stockpiles of cash, and instead distribute all excess cash holdings at the end of each period to their owners in the household sector, keeping their net worth at zero. Since real assets can change in value, maintaining the symmetry principle requires that loans adjust in response to a change in the value of a real asset. Since for every

Table 1: The balance sheet matrix tabulates all stocks in monetary values. The money deposits of households M_h are equivalent to the money issued by the government M_g, because the industry sector does not hold money deposits. The loans L_i of each sector i are equal to the unit cost inventories Ψ_i. They sum up to the total outstanding loans of the government / banking sector L_g. Since financial assets and liabilities within the economy sum up to zero, the net worth of the system as a whole is equal to the value of inventories (the only real asset). All sums over i are proceeded over the n industry sectors.

	Households	**Government**	**Industry** $\boldsymbol{i \in \{1, ..., n\}}$	**line total**
Money Deposits	$+M_h$	$-M_g$		0
Loans		$+L_g$	$-L_i$	0
Inventories			$+\Psi_i$	$+\sum_i \Psi_i$
Net worth	$-V_h$	$-V_g$	0	$-V_h - V_g$
$\sum$	0	0	0	0

financial asset in the economy there is a corresponding financial liability, the net worth of the model as a whole consists only of the monetary values of real assets (inventories), because all financial assets and financial liabilities must necessarily sum to zero.

The very existence of stocks introduces historical time and a certain path dependence into the model. Even though the model may asymptotically converge to a steady state if all exogenous parameters are undisturbed, the model will follow a different traverse path for every possible set of stocks. Moreover, not all conceivable sets of stocks are in fact possible; only some sets of stocks are consistent with the model's accounting. Thus, depending upon the set of stocks with which the model economy has been endowed by the past, the model will follow a different trajectory forwards into the future.

The accounting identities in table 1 for the stocks hold for all time periods t, where the sum of i is calculated over all n industry sectors:

$$V_{h(t)} = M_{h(t)}, \tag{6}$$

$$V_{g(t)} = L_{g(t)} - M_{g(t)}, \tag{7}$$

Table 2: The Transaction matrix tabulates all flows of funds within one time period in monetary values. The fact that the columns sum to zero represents a sector's budget constraints, while the fact that rows sum to zero represents the fact that each financial transaction has a counterparty. Positive values indicate inflows, while negative values indicate outflows. For example, taxes paid by households to the government are an outflow from households and an inflow to the government, so $-T$ appears in the household sector column and $+T$ appears in the government sector column. In the flow of funds accounts, outflows are referred to as *uses* of funds, whereas inflows are referred to as *sources* of funds. Note, however, that if households increase their holdings of money (a change in a stock), this is a *use* of funds even though there is no outflow. So for changes in stocks, the fact that sources of funds are denoted with a plus sign and uses of funds are denoted with a negative sign can seem counterintuitive. To clarify this, consider that a positive increase in money balances constitutes a use of funds, not a source. For example, if the household sector increases its money balances, it is using its funds to accumulate a stock of money, as opposed to using its funds for another purpose such as consumption. The current account includes receipts and outlays of firms, whereas the capital account includes capital expenditures (investment) (Godley and Lavoie, 2007).

	Households	**Industry i**		**Government**	$\sum$
		Current acc.	**Capital acc.**		
Government spending		$+G_i$		$-\sum_i G_i$	0
Taxes	$-T$			$+T$	0
Consumption	$-\sum_i C_i$	$+C_i$			0
Wage bill	$+\sum_i Ш_i$	$-Ш_i$			0
Intermediate purchases		$\sum_i E_{ij} - \sum_j E_{ij}$			0
Profits	$+\sum_i \Pi_i$	$-\Pi_i$			0
Interest on money deposits	$+r_M M_{h(t-1)}$			$-r_M M_{g(t-1)}$	0
Interest on loans		$-r_L L_{i(t-1)}$		$+\sum_i r_L L_{i(t-1)}$	0
Δ Money deposits	$-\Delta M_h$			$+\Delta M_g$	0
Δ Loans			$+\Delta L_i$	$-\sum_i \Delta L_i$	0
Δ Inventory Value		$+\Delta\Psi_i$	$-\Delta\Psi_i$		0
$\sum$	0	0	0	0	0

$$M_{h(t)} = M_{g(t)}, \tag{8}$$

$$L_{g(t)} = \sum_i L_{i(t)}, \tag{9}$$

$$L_{i(t)} = \Psi_{i(t)}, \qquad \Delta L_i = L_{i(t)} - L_{i(t-1)} \tag{10}$$

$$V_{h(t)} + V_{g(t)} = \sum_i \Psi_{i(t)}. \tag{11}$$

The flows shown in figure 1 are also displayed in a more general representation with n sectors in the transaction matrix in table 2. Adherence to the accounting constraints imposed by the balance sheet in table 1 and the transaction-flow matrix in table 2 guarantees the consistency of the model, and can be verified by checking that all the columns and rows of the matrices sum to the appropriate values, which is zero in the case of financial assets (Godley and Lavoie, 2007, p. 27). All parameters are summarized in table 3. In the following, all matrices are displayed as bold roman letters, vectors in bold italic characters. $\mathbf{diag}(x_i)$ indicates a diagonal matrix with x_i on the diagonal.

3.1 Banking Sector

Unlike most SFC models, the role of the banking sector (which is consolidated as a simplification into the government sector along with the central bank) is very limited here. Banks provide loans to industries whenever requested and can increase the money stock via credit creation. Though in truth banks may ration credit to industries, loans in this simple model are provided to the industry sectors on demand (Moore, 1988; Wray, 1990). There are two interest rates in the model. First, there is an interest rate on loans r_L paid by each industry sector to the government on the stock of loans from the previous period $L_{i(t-1)}$. Second, there is an interest rate on money deposits r_M paid by the government to households on the stock of money deposits from the previous period $M_{h(t-1)}$.

3.2 Household Sector

Households are treated as an aggregated sector and hold only one type of financial asset: money deposits. The only behavioral decision of households in this model is consumption. It is assumed that in the aggregate, a certain fraction α_1 of the wage bill after taxes $(1-\theta)Ш$ (with θ: tax rate), and a smaller fraction α_2 of wealth $M_{h(t-1)}$ is consumed, see Godley and Lavoie (2007, p. 66) for a justification of a similar consumption function. Interest payments and distributed profits are not used as sources of finance for consumption within the period, but rather are added to wealth in the current period. In subsequent periods, a portion of this accumulated stock of financial wealth will be used to finance consumption. In this manner, a smaller propensity to consume from capital income compared to wage income is guaranteed. Total consumption C is allocated to consumption goods produced by each individual industry sector by an exogenous vector $\boldsymbol{C^0}$:

$$C = \alpha_1(1-\theta)\text{Ш} + \alpha_2 M_{h(t-1)}, \tag{12}$$

$$C_i = CC_i^0 \leq C \qquad \text{with} \sum\nolimits_j C_j^0 = 1. \tag{13}$$

Once prices P_i are set by the industry sectors, the real physical demand c_i of the households can be calculated as:

$$c_i = \frac{C_i}{P_i} \qquad \forall\ i. \tag{14}$$

The money stock M_h held by households is increased by incoming flows Y consisting of the wage bill $\text{Ш}=\sum_i \text{Ш}_i$ paid to households by the industry sectors, the profits $\sum_i \Pi_i$ distributed to households by the industry sectors, and interest on money deposits paid to households by the banking system $r_M M_{h(t-1)}$. The money stock M_h is decreased by consumption of goods and services from the industry sectors $\sum_i C_i$ and by taxes T that are levied as a constant share $\theta < 1$ of income Y:

$$Y = \sum\nolimits_i \text{Ш}_i + \sum\nolimits_i \Pi_i + r_M M_{h(t-1)}, \tag{15}$$

$$M_{h(t)} = (1+r_M)M_{h(t-1)} + \sum\nolimits_i (\text{Ш}_i + \Pi_i - C_i) - T, \tag{16}$$

$$T = \theta \cdot Y. \tag{17}$$

3.3 Government Sector

We assume that nominal government spending on the output of each sector is exogenously given by G_i, with total expenditure $G = \sum_i G_i$. The real physical demand of the government can be calculated as:

$$g_i = \frac{G_i}{P_i} \qquad \forall\ i. \tag{18}$$

The money stock M_g issued by the government is increased by outflows of government spending $\sum_i G_i$, by the increase $\sum_i \Delta L_i$ in the outstanding stock of loans, and by interest payments on money deposits $r_M M_{g(t-1)}$. It is decreased by inflows of tax payments T and of interest payments on loans $\sum_i r_L L_{i(t-1)}$.

$$M_{g(t)} = (1 + r_M) M_{g(t-1)} + \sum\nolimits_i (G_i + \Delta L_i - r_L L_i) - T. \tag{19}$$

As mentioned in section 3, the money stock M_g issued by the government is always equal to the money stock M_h held by households because of accounting consistency spelled out in the transactions flow matrix.

3.4 Industry Sectors

In contrast to most SFC models, in which the production sector is highly aggregated, the interlinkages between sectors are articulated using an IO model. An economy with n sectors is described by an $n \times n$ IO matrix of technical coefficients $\mathbf{a} = (a_{ij})$ given in physical terms. If $a_{ij} > 0$, the production of good j by industry j requires a physical flow of inputs of good i from industry i to industry j, such that a_{ij} units of input i are required to produce each unit of output j. The prices of the goods are contained in the diagonal matrix $\mathbf{P}$. In general, real magnitudes can be converted to nominal magnitudes by multiplying by current prices.

$$\mathbf{a} = (a_{ij}), \tag{20}$$

$$\mathbf{P} = \mathbf{diag}(P_i) \qquad \Leftrightarrow \qquad P_{ij} = P_i \delta_{ij}. \tag{21}$$

with δ_{ij} being the Kronecker delta that is 1 iff $i = j$ and 0 otherwise.

The IO matrix can also be viewed in monetary terms by multiplying $\mathbf{P}$ by $\mathbf{a}$, yielding matrix $\mathbf{A}$. Note that this illustrates our convention that upper case letters refer to nominal monetary values, while lower case letters refer to real values.

$$\mathbf{A} = \mathbf{P} \cdot \mathbf{a} \qquad \Leftrightarrow \qquad A_{ij} = a_{ij} P_i. \tag{22}$$

The total real quantity of goods sold $\boldsymbol{s_{(t)}}$ will consist of sales to the households $\boldsymbol{c}$, sales of intermediate inputs to other industries $\boldsymbol{\xi}$, and sales to the government $\boldsymbol{g}$. Note that because sales of intermediate inputs (which in value-added terms net to zero) are included in the definition of sales, these are not *net* sales, but rather are *gross* sales.

$$\boldsymbol{s_{(t)}} = \boldsymbol{c} + \boldsymbol{\xi} + \boldsymbol{g}. \tag{23}$$

The sales in the model economy are not determined on price-clearing auction markets. As in the real world, realized sales are not yet known at the time when production decisions are made, so each industry i must estimate its *expected* sales $\boldsymbol{s^X}$ with a partial adjustment model, adjusting expectations partially $(0 < \beta < 1)$ from each period to the next:

$$\boldsymbol{s^X_{(t)}} = \beta \boldsymbol{s_{(t-1)}} + (1 - \beta)\, \boldsymbol{s^X_{(t-1)}}. \tag{24}$$

As a buffer against unexpected changes in sales, firms build up stocks of inventories ψ_i, much as do retail stores in the real world (Clower, 1965; Godley and Lavoie, 2007; Hicks, 1989; Leijonhufvud, 1968). The inventory target $\boldsymbol{\psi}^\top$ is considered as a fraction of expected sales $\boldsymbol{s^X}$, and is also updated with a partial accelerator function in light of the experiences of the previous period (with $0 < \gamma < 1$). $\sigma^\top > 0$ is the targeted ratio of inventories to expected sales. This leads to a demand for inventories $\boldsymbol{\Delta\psi}^\top$

$$\boldsymbol{\psi}^\top = \sigma^\top \boldsymbol{s^X_{(t)}}, \tag{25}$$

$$\boldsymbol{\Delta\psi}^\top = \gamma \left[\boldsymbol{\psi}^\top - \boldsymbol{\psi_{(t-1)}}\right]. \tag{26}$$

The industry sectors produce a total gross output $\boldsymbol{x}$ in physical terms, which is equal to the total amount of expected sales plus the targeted change in inventory stocks by the industry itself. Note that expected *gross* sales includes expected sales of intermediate inputs as well as expected sales for final demand:

$$\boldsymbol{x} = \boldsymbol{s^X_{(t)}} + \boldsymbol{\Delta\psi}^\top_{(t)}. \tag{27}$$

This production of output requires a labor force $\boldsymbol{l}$. In each sector, the required labor force is assumed to be proportional to gross production x_i of the sector:

$$l_i = \lambda_i x_i, \tag{28}$$

where the vector $\boldsymbol{\lambda}$ contains the labor forces λ_i required for production in a specific sector. Given the wages per labor unit ω_i, the wage bills paid per sector $\text{Ш}_i = \omega_i \lambda_i x_i$ add up to:

$$\text{Ш} = \sum_i \text{Ш}_i = \sum_i \omega_i \lambda_i x_i. \tag{29}$$

Because intermediate goods are consumed in production, gross product always exceeds net product bound for final delivery. Since gross output $\boldsymbol{x}$ is already known, net output $\boldsymbol{d}$ can be calculated. Though we are solving for net output using a given gross output, rather than solving for gross output using a given net output, this does not mean that effective demand is unimportant. Indeed, *expectations* of net output (effective demand) play a crucial causal role in driving the model. Given expectations of net output, or for that matter expectations of gross output, entrepreneurs decide how much gross output to actually produce. Given this *realized* gross output, one can derive the net output $\boldsymbol{d}$ including inventory production:

$$\boldsymbol{d} = (\mathbf{1} - \mathbf{a})\, \boldsymbol{x}. \tag{30}$$

Using standard IO analysis, the sales of intermediate inputs (in real terms) to other industries can be calculated as:

$$\boldsymbol{\xi} = \mathbf{a} \cdot \boldsymbol{x}. \tag{31}$$

In reality, firms must use costing procedures to estimate their costs, and firms will often set a markup over some form of estimated normal costs, but the difficulties of cost estimation are abstracted from here. Post-Keynesian economists have often assumed constant prime or direct costs (Eichner and Kregel, 1975, p. 1305). However, there is empirical evidence that average direct costs may increase or decrease as output increases, and that cost structures vary across different industries (Lee, 1986). We assume that the price for each sector is set as a markup ϕ_i on unit costs from the previous time period. Costs in this model include wages and the costs of intermediate inputs, causing prices to be interdependent, because prices of other goods alter production costs:

$$P_{i(t)} = \left(1 + \phi_{i(t)}\right) \left[\omega_{i(t-1)}\lambda_{i(t-1)} + \sum\nolimits_k P_{k(t-1)} a_{ki(t-1)}\right]. \tag{32}$$

Given the gross production vector $\boldsymbol{x}$, the monetary flows for intermediate inputs can be calculated as a matrix $\mathbf{E}$, where E_{ij} means a flow of money from industry j to industry i:

$$\mathbf{E} = \mathbf{P}\ \mathbf{a}\ \mathbf{diag}(x_i) \qquad \Leftrightarrow \qquad E_{ij} = a_{ij} P_i x_j. \tag{33}$$

The realized inventories $\boldsymbol{\psi_{(t)}}$ at the end of the period then depend on realized sales $\boldsymbol{s_{(t)}}$. Because there can be a discrepancy between expected sales $\boldsymbol{s^X_{(t)}}$ and realized sales $\boldsymbol{s_{(t)}}$, the realized amount of inventories $\boldsymbol{\psi_{(t)}}$ may differ from the expected. The monetary value of inventories $\boldsymbol{\Psi}_{(t)}$ is equal to the physical quantity of inventory units multiplied by the unit cost of inventories, which includes wage costs and intermediate input costs:

$$\boldsymbol{\psi_{(t)}} = \boldsymbol{\psi_{(t-1)}} + \boldsymbol{x_{(t)}} - \boldsymbol{s_{(t)}}, \tag{34}$$

$$L_{i(t)} = \Psi_{i(t)} = \psi_{i(t)} \left[\omega_{i(t-1)}\lambda_{i(t-1)} + \sum\nolimits_k P_{k(t-1)} a_{ki(t-1)}\right]. \tag{35}$$

The net profit of each industry $\mathbf{\Pi}$ has two components: The monetary profit of industry i is the sum of households' consumption expenditures, purchases by government, and intermediate investment E_{ij} by the other sectors of the economy minus intermediate purchases E_{ji} of sector i and minus the wage bill $Ш_i$; additionally, interest payments $r_L L_{i(t-1)}$ must be subtracted to get net profits. The second component contributing to profits is any change of the value of the inventories $\Delta\Psi_i = \Psi_{i(t)} - \Psi_{i(t-1)}$, valued at current unit costs. Note that profits are a residual determined by accounting constraints; the equation for profits can simply be read off of the industry i current account column on the transaction flow matrix in table 2.

$$\Pi_i = C_i + G_i - Ш_i + \sum\nolimits_j E_{ij} - \sum\nolimits_j E_{ji} - r_L L_{i(t-1)} + \Delta\Psi_i. \tag{36}$$

We assume that the industrial sectors distribute all profits to the households.

Table 3: Parameters in the general model and the values used in section 5. For simplicity, ω_i and λ_i were merged into one single parameter.

parameter name	**model presented in section 5**
Consumpt. func. parameters: α_1, α_2	$\alpha_1 = 0.8, \alpha_2 = 0.2$
Input-Output matrix: $\mathbf{a} = (a_{ij})$	$\mathbf{a} = \begin{bmatrix} 0.48 & 0.60 \\ 0.02 & 0.15 \end{bmatrix}$
Price matrix: $\mathbf{P} = \mathbf{diag}(P_i)$	$\mathbf{P} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Partial adjustment accelerators: β, γ	$\beta = 0.75, \gamma = 0.5$
Government spending: $\boldsymbol{G}$	$G_p = 46.6$, $G_e = 0$
Consumption per sector: $\boldsymbol{C^0}$	$C_p^0 = 0.961$, $C_e^0 = 0.039$
Individual markups: $\boldsymbol{\phi}$	$\phi_p = 0.3333$, $\phi_e = 0.1364$
Interest rates: r_M, r_L	$r_M = 0.04$, $r_L = 0.05$
Tax rate: θ	$\theta = 0.48$
Inventory to expected sales ratio: $\sigma^\top$	$\sigma^\top = 0.5$
Labor demand per output unit: $\boldsymbol{\lambda}$ Wages per labor unit: $\boldsymbol{\omega}$	$\omega_p\lambda_p = 0.25$; $\omega_e\lambda_e = 0.13$

3.5 Solving the Model

Usually, SFC models contain implicit functions and are typically solved numerically by iterative techniques (Caverzasi and Godin, 2013; Godley and Lavoie, 2007), but in this case, the time step evolution can be solved explicitly, though because of the number of variables, the calculations will be performed numerically. All relevant parameters are put together in table 3.

From the last period, the stocks of money $M_{h(t-1)}$ and loans $L_{i(t-1)}$ and the corresponding values for the government, the inventories $\boldsymbol{\psi_{(t-1)}}$, the prices $P_{i(t-1)}$ and the expected and realized sales $\boldsymbol{s^X_{(t-1)}}$ and $\boldsymbol{s_{(t-1)}}$ are known, together with the IO matrix $\mathbf{a}$. The prices will be updated using equation (32), the expected sales and targeted inventory adjustments will be calculated using (24) and (26), and the total production using (27). Then, the wage bill can be calculated using (29), and the physical demand of the households and government can be calculated using equations (12), (13), (14) and (18). This being known, the realized inventories ψ_i are given by (34), the monetary value of inventories at unit costs Ψ_i and therefore the loans to finance them by (35). Additionally, the realized intermediate sales $\boldsymbol{\xi}$ (31) and gross sales $\boldsymbol{s_{(t)}}$ (23) can be determined. The payments within the sector can be calculated using (33), the distributed profits using (36), and taxes using (17), the monetary stocks at the end of the period using (16) for the households, (19) for the government. In this way, all new stocks can be calculated straightforwardly, without any iterative procedure.

It can be stated that the model dynamics can be decomposed into two procedures: The price adjustment mechanism and the dynamics of the economy. It is crucial to point out that the pricing depends only on the input–output matrices, the wages and the markup (and later the interest costs). These factors are independent on the actual scale or dynamics of the rest of the economy.

The pricing equation (32) in matrix form, assuming that ϕ_i, ω_i, λ_i and a_{ki} are constant over time, is given by:

$$\boldsymbol{P_{(t)}} = \boldsymbol{P_{(t-1)}}\mathbf{a}(\mathbf{1} + \mathbf{diag}(\phi_i)) + \mathbf{diag}((1 + \phi_i)\lambda_i\omega_i). \tag{37}$$

If prices are calculated, the evolution of the other economic variables is defined by an non-homogeneous first-order matrix difference equation. In the special case of n=2, thus for an economy with two sectors denoted p and e, the time evolution is given by the map $\mathcal{M}$:

$$\boldsymbol{X}_{(t)} = \begin{pmatrix} s^X_{p(t)} \\ s^X_{e(t)} \\ s_{p(t)} \\ s_{e(t)} \\ \psi_{p(t)} \\ \psi_{e(t)} \\ M_{h(t)} \end{pmatrix} = \mathcal{M} \cdot \boldsymbol{X}_{(t-1)} + \begin{pmatrix} 0 \\ 0 \\ +G_p/P_{p(t)} \\ +G_e/P_{e(t)} \\ -G_p/P_{p(t)} \\ -G_e/P_{e(t)} \\ 0 \end{pmatrix}, \qquad \text{with } \mathcal{M} = \tag{38}$$

$$\begin{bmatrix} 1-\beta & 0 & \beta & 0 & 0 & 0 & 0 \\ 0 & 1-\beta & 0 & \beta & 0 & 0 & 0 \\ Z_{pp}\mathrm{L} & Z_{pe}\mathrm{L} & Z_{pp}\Gamma & Z_{pe}\Gamma & -Z_{pp}\gamma & -Z_{pe}\gamma & \alpha_2 C^0_p/P_{p(t)} \\ Z_{ep}\mathrm{L} & Z_{ee}\mathrm{L} & Z_{ep}\Gamma & Z_{ee}\Gamma & -Z_{ep}\gamma & -Z_{ee}\gamma & \alpha_2 C^0_e/P_{e(t)} \\ (1-Z_{pp})\mathrm{L} & -Z_{pe}\mathrm{L} & (1-Z_{pp})\Gamma & -Z_{pe}\Gamma & 1-(1-Z_{pp})\gamma & Z_{pe}\gamma & -\alpha_2 C^0_p/P_{p(t)} \\ -Z_{ep}\mathrm{L} & (1-Z_{ee})\mathrm{L} & -Z_{ep}\Gamma & (1-Z_{ee})\Gamma & Z_{ep}\gamma & 1-(1-Z_{ee})\gamma & -\alpha_2 C^0_e/P_{e(t)} \\ Z_p\mathrm{L} & Z_e\mathrm{L} & Z_p\Gamma & Z_e\Gamma & r_L P_{p(t)}\frac{\theta-1}{1+\phi_p}-Z_p\gamma & r_L P_{e(t)}\frac{\theta-1}{1+\phi_e}-Z_e\gamma & 1+r_M(1-\theta)-\alpha_2 \end{bmatrix},$$

using the following definitions:

$$Z_{ij} = a_{ij} + \alpha_1(1-\theta)\omega_j\lambda_j C^0_i \qquad \text{with} \qquad i,j \in p,e, \tag{39}$$

$$Z_p = (1-\theta)(P_{p(t)} - a_{pp}P_{p(t)} - a_{ep}P_{e(t)} - \alpha_1\omega_p\lambda_p), \tag{40}$$

$$Z_e = (1-\theta)(P_{e(t)} - a_{pe}P_{p(t)} - a_{ee}P_{e(t)} - \alpha_1\omega_e\lambda_e), \tag{41}$$

$$\Gamma = (1+\gamma\sigma^\top)\beta, \tag{42}$$

$$\mathrm{L} = (1+\gamma\sigma^\top)(1-\beta). \tag{43}$$

Z_{ij} can be associated with the income of sector i generated by sector j via intermediate purchases or wage consumption of its workers. Z_i corresponds to the non-consumed, non-taxed income of households by sector i. All Z_{ij} lie between 0 and 1. If we assume that a stable equilibrium price vector exists and prices have converged to that vector, this difference equation governs the behavior of the dynamical system.

If prices are renormalized to $P_p = P_e = 1$, all Z_i lie between 0 and 1, and the following equations hold:

$$\begin{aligned} Z_p &= (1-\theta)(1-a_{pp}-a_{ep}-\alpha_1\omega_p\lambda_p) \\ &= (1-\theta)\left(\frac{\phi_p}{1+\phi_p}+(1-\alpha_1)\omega_p\lambda_p\right) \\ &= (1-\theta)(1-Z_{pp}-Z_{ep}-\alpha_1\theta\omega_p\lambda_p). \end{aligned} \tag{44}$$

To calculate the other stocks and flows existing in the model, first calculate total production by

$$x_p = \left[(1+\gamma\sigma^\top)(\beta s_{p(t-1)}+(1-\beta)s^X_{p(t-1)})-\gamma\psi_{p(t-1)}\right], \tag{45}$$

$$x_e = \left[(1+\gamma\sigma^\top)(\beta s_{e(t-1)}+(1-\beta)s^X_{e(t-1)})-\gamma\psi_{e(t-1)}\right], \tag{46}$$

and continue with the procedure explained above.

The economy reaches a general stationary stock-flow equilibrium if all stocks and all flows remain constant over time, and therefore inflows equal outflows. The equilibrium value $\boldsymbol{X}^*$ can be determined by calculating

$$\boldsymbol{X}^* = (\mathbf{1}-\mathcal{M})^{-1}\cdot\begin{pmatrix} 0 \\ 0 \\ +G_p/P_p \\ +G_e/P_e \\ -G_p/P_p \\ -G_e/P_e \\ 0 \end{pmatrix}. \tag{47}$$

The unique stock-flow equilibrium is a stable fixed point if the absolute values of all eigenvalues of the mapping matrix $\mathcal{M}$ are smaller than 1.

4 Stability Analysis

An important part of this thesis that goes beyond the paper (Berg et al., 2015b) is a rigorous stability analysis of the model by means of dynamical system theory. As shown in section 3.5, the dynamics can be decomposed into the pricing process and the rest of the economic process. This is caused by the somewhat unrealistic assumption of an exogenous markup. If the markup were endogenized and various economic forces such as competition, market power as well as the effect of unemployment on worker's bargaining power were taken into account endogenously, the dynamics could not be broken up into two pieces this way.

The matrix of the time evolution in equation (38) has two eigenvalues that are 0, two pairs of complex eigenvalues, and one real eigenvalue > 0. The matrix is therefore singular and not invertible. The zero eigenvalues are caused by the adaption process of expected sales displayed in equation (24). The linear subspace of the corresponding stable manifold exists for all parameter values and is given by

$$\Big(\mu_1\beta, \mu_2\beta, \mu_1(1-\beta), \mu_2(1-\beta), 0, 0, 0\Big)^{\intercal} \text{ for } \mu_1, \mu_2 \in \mathbb{R}. \tag{48}$$

Therefore, the system is non-conservative, as $J=0$ as defined by equation 5.

The real, positive eigenvalue is studied in section 4.2 in the context of an instability caused by the interaction between the interest rate and the propensity to consume. The complex eigenvalues are studied in section 4.3 on inventory oscillations. Studying the stability of the pricing process yields a generalized version of the Sraffian maximum rate of profit in section 4.1. In the following, if no other values are indicated, the parameters from table 3 are used.

4.1 Price Interdependence and the Sraffian Maximum Rate of Profit

In the model, pricing is determined by a sector-specific markup ϕ_i on unit costs which consist of the unit wage bill $\omega_i \lambda_i$ and the intermediate purchases $\sum_k P_{k(t-1)} a_{ki(t-1)}$. In matrix form, assuming that ϕ_i, ω_i, λ_i and a_{ki} are constant over time, the price evolution yields:

$$\boldsymbol{P_{(t)}} = \boldsymbol{P_{(t-1)}} \mathbf{a}(\mathbf{1} + \mathbf{diag}(\phi_i)) + \mathbf{diag}((1 + \phi_i)\lambda_i \omega_i). \tag{49}$$

Thus if the absolute value of one of the eigenvalues of matrix $\mathbf{a}(\mathbf{1} + \mathbf{diag}(\phi_i))$ is greater than 1, prices do not converge to stable values, but rather explode. The passing of the threshold corresponds to a transcritical bifurcation, the bounded price equilibrium loses its stability, and the price trajectory is unboundedly going to infinity.

In the general case with n sectors, this would correspond to a $n-1$-dimensional stability hyperspace in n-dimensional space. In the case of n=2 sectors p and e, we can calculate the maximum markup in one sector dependent on the markup in the other sector and draw a one-dimensional stability frontier, as shown in figure 2:

$$\phi_p^{max} = \frac{1 - (1 + \phi_e) a_{ee}}{a_{pp} - (1 + \phi_e)(a_{pp} a_{ee} - a_{ep} a_{pe})} - 1, \tag{50}$$

$$\frac{\partial \phi_p^{max}}{\partial \phi_e} = \frac{-a_{ep} a_{pe}}{\left[(1 + \phi_e)(a_{pp} a_{ee} - a_{ep} a_{pe}) - a_{pp}\right]^2} \leq 0. \tag{51}$$

If the sectors are not interconnected and $a_{ep} a_{pe}$=0, $\phi_p^{max} = a_{pp}^{-1} - 1$ is independent of ϕ_e. In all other cases, the value of ϕ_p^{max} is maximized if ϕ_e=0 and then yields:

$$\phi_p^{max} = \frac{1}{a_{pp} + \frac{a_{ep} a_{pe}}{1 - a_{ee}}} - 1. \tag{52}$$

We can now study the impact of this price inflation on the economy. If the nominal payment G of the government is fixed, then ever increasing prices will drive down the real production of the economy, while at the same time the nominal wealth of households and goverment debt grow without limit. If government expenditures are price adjusted, the economy stabilizes at a lower real level, see figure 3. In both cases, the profit share approaches 1.

This corresponds to the Sraffian maximum rate of profit, 'the rate of profits as it would be if the whole of the national income went to profits' (Sraffa, 1960, p. 19). This means that if the markup is set higher than the maximum real rate of profit, the price system will adjust so that the whole of the national income goes to profits as defined by Sraffa. In Sraffa's case, the profit ϕ_{max} was identical for all sectors and the maximum rate of profit was given by

$$\phi_{max} = (1/\lambda_a^{max}) - 1, \tag{53}$$

with λ_a^{max} being the maximum of the moduli of the eigenvalues of the input–output matrix $\mathbf{a}$ (Eatwell, 1975). The price instability is a generalized version of the Sraffian maximum rate of profit in the case of different markups in each sector: It's about finding a *vector* of heterogeneous maximum rates of profit, while in Sraffa's original formulation, the vector is assumed to be ϕ_{max} times an all-ones vector.

Though this generalization is very straightforward, a literature review did not yield any publication mentioning this result. The work of Sraffa incorporates a uniform profit rate condition, because competition is assumed to equalize profits (Lawlor, 1994). Therefore, work in the tradition of Sraffa mostly assume identical markup in each sector of the economy, which may be part of the reason why heterogenous markups have not been more prominent in Sraffian literature.

4.2 The Instability of a Stationary Economy with Positive Interest Rates

Within ecological economics, several authors propose a non-growing economy as a solution to environmental problems (Daly, 1991; Jackson, 2009; Kallis et al., 2012; Pueyo, 2014). In recent publications, it has been claimed that this is incompatible with positive interest rates (Farley et al., 2013; Löhr, 2012). It is argued that positive interest rates imply that in a non-growing economy, the stock of debts will rise, and it is argued that such an increase would be unsustainable. Using our model, we show that an equilibrium state of a stationary economy is possible, even with positive interest rates. To facilitate the analysis, we assume that $P_p = P_e = 1$. The stability of the

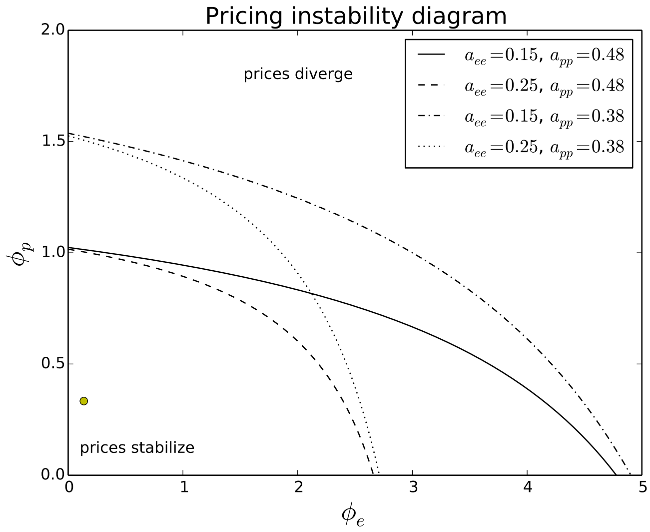

Figure 2: Pricing instability for $n = 2$ sectors: The lines correspond to the stability frontier of different input–output matrices, given by the eigenvalues of the matrix in equation 49 passing the unit circle. The black dot indicates the position of the markups given in table 3 estimated for the German economy from (Statistisches Bundesamt, 2010b), which is well within the stable region for the input–output coefficients used that are indicated by the solid line.

stock-flow equilibrium is graphed in the parameter space of interest rates $r_{M/L}$, consumption parameters $\alpha_{1/2}$, and for different tax rates θ in figures 4 and 5. The stability frontiers depicted correspond to the real non-zero eigenvalue of matrix $\mathcal{M}$ passing the unit circle at 1. It is a transcritical bifurcation, where the bounded fixed point loses its stability, and the time evolution start to be divergent, while the fixed point of the map becomes undefined if the eigenvalue is 1 and changes sign at the bifurcation point according to equation 47.

Complementing the purely numerical results published in Berg et al. (2015b), an analytical solution for the stability frontier can be calculated: If one the eigenvalues of the 7×7 matrix in equation (38) is 1, then the corresponding eigenvector $X'_{(t)}$ has the following property:

$$s'^{X}_{p(t)} = s'_{p(t)}, \qquad s'^{X}_{e(t)} = s'_{e(t)}, \qquad \phi'_{p(t)} = s'^{X}_{p(t)}, \qquad \phi'_{e(t)} = s'^{X}_{e(t)}. \tag{54}$$

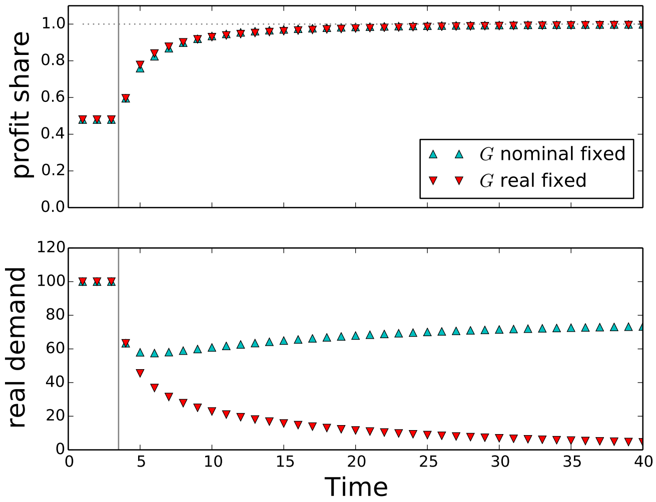

Figure 3: Time evolution profit share and real demand in the case of markups above the generalized Sraffian maximum rate of profit, therefore outside the stability range of figure 2. The values depicted are $\phi_p = 1.1$, $\phi_e = 1.0$, while $\gamma = 0.80$ and $\beta = 0.02$ used in order to smoothen the graph. The graphs depicts two behavioral assumptions, keeping government expenditures G fixed in nominal terms with real output of the economy declining to zero, and price adjusted expenditures resulting in a stabilization of real output at a lower equilibrium value. In both cases, the profit share converges to 1 as predicted.

In this case, one can replace the time evolution with this 3×3 matrix equation:

$$\boldsymbol{X'_{(t)}} = \begin{pmatrix} s'_{p(t)} \\ s'_{e(t)} \\ M'_{h(t)} \end{pmatrix} = \mathcal{M}' \cdot \boldsymbol{X'_{(t-1)}} + \begin{pmatrix} +G_p \\ +G_e \\ 0 \end{pmatrix}, \qquad \text{with } \mathcal{M}' = \tag{55}$$
$$\begin{bmatrix} Z_{pp} & Z_{pe} & \alpha_2 C_p^0 \\ Z_{ep} & Z_{ee} & \alpha_2 C_e^0 \\ Z_p + \sigma^\top r_L \frac{\theta-1}{1+\phi_p} & Z_e + \sigma^\top r_L \frac{\theta-1}{1+\phi_e} & 1+r_M(1-\theta)-\alpha_2 \end{bmatrix}.$$

One root of the eigenvalue polynomial of this matrix has to be 1 following our assumption, and the equation can then be solved for α_2 to determine the

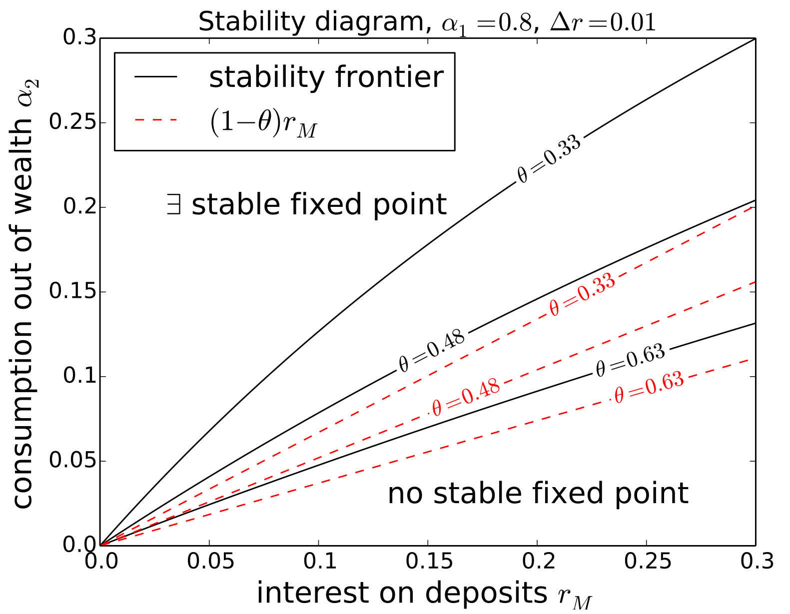

Figure 4: Stability diagram for the interdependence of interest rate r_M and consumption parameter α_2, including the influence of the tax rate θ. For different tax rates θ, we check whether a stable stock-flow equilibrium exists. For a given interest rate r_M, there exists a minimum consumption out of wealth α_2 for which the model is stable, given by equation 57. An increase in the tax rate reduces this threshold. If consumption out of wealth is smaller than interest income after taxes (as indicated by the dashed lines), the fixed point will definitely be unstable, as inflows to households are always bigger than outflows for $\alpha_1 < 1$.

minimal consumption rate out of wealth:

$$\alpha_2 = \frac{r_M(1-\theta)\left[Z_{ep}Z_{pe} - (Z_{pp}-1)(Z_{ee}-1)\right]}{\begin{array}{l} Z_{ep}Z_{pe} - (Z_{pp}-1)(Z_{ee}-1) \\ \quad + \left(Z_{ep}C_p^0 - (Z_{pp}-1)C_e^0\right)\left(Z_e + \sigma^\top r_L \frac{\theta-1}{1+\phi_e}\right) \\ \quad\quad + \left(Z_{pe}C_e^0 - (Z_{ee}-1)C_p^0\right)\left(Z_p + \sigma^\top r_L \frac{\theta-1}{1+\phi_p}\right) \end{array}}, \tag{56}$$

which can be reformulated using the definition of Z_p and Z_e and the fact that $C_e^0 + C_p^0 = 1$ to get:

$$\alpha_2 = \frac{r_M\left[(1-Z_{pp})(1-Z_{ee}) - Z_{ep}Z_{pe}\right]}{\begin{array}{l} \frac{\theta}{1-\theta}\left[(1-Z_{pp})(1-Z_{ee}) - Z_{ep}Z_{pe}\right] \\ \quad + \left(Z_{ep}C_p^0 + (1-Z_{pp})C_e^0\right)\left(\frac{\sigma^\top r_L}{1+\phi_e} + \alpha_1\theta\omega_e\lambda_e\right) \\ \quad\quad + \left(Z_{pe}C_e^0 + (1-Z_{ee})C_p^0\right)\left(\frac{\sigma^\top r_L}{1+\phi_p} + \alpha_1\theta\omega_p\lambda_p\right) \end{array}}. \tag{57}$$

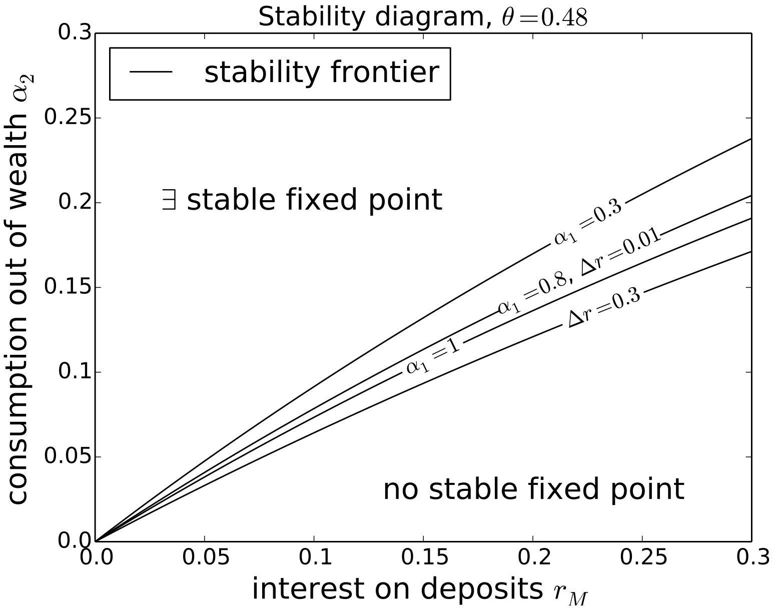

Figure 5: Stability diagram for the influence of interest rate spread and consumption out of wages. The impact of the interest rate spread $\Delta r = r_L - r_M$ and the consumption parameter α_1 are depicted, $\alpha_1 = 0.8$ and $\Delta r = 0.01$ serves as a benchmark; only changes of these parameters are indicated. A higher interest rate spread shifts the stability lines down slightly. A higher consumption out of wages α_1 increases the size of the stable region.

The term

$$(1 - Z_{pp})(1 - Z_{ee}) - Z_{ep}Z_{pe} \tag{58}$$

corresponds to the determinant of $\mathbf{1} - \boldsymbol{Z}$ which is negative only if one of the eigenvalues of $\boldsymbol{Z}$ is bigger than 1. This cannot be the case, as $a_{ii} + a_{ij} + \omega_i\lambda_i$ must be always smaller than 1 in order to guarantee a positive markup. As $0 < Z_{ij} < 1$ and all other parameters are positive, α_2 is always a well-defined non-negative number. The result is independent on government expenditures $\boldsymbol{G}$.

In the special case of $a_{pp} = a_{ee}$, $a_{ep} = a_{pe}$, $\omega_p\lambda_p = \omega_e\lambda_e = \omega\lambda$ and therefore $\phi_p = \phi_e = \phi$, thus in case of symmetrical production conditions, even with $C_p^0 \neq C_e^0$ as the two commodities are structurally identical, this simplifies to:

$$\alpha_2 = r_M(1-\theta)\frac{\phi + \omega\lambda(1+\phi)(1-\alpha_1(1-\theta))}{\theta(\phi + \omega\lambda(1+\phi)) + \sigma^\top r_L(1-\theta)}. \tag{59}$$

We can see that α_2 is proportional to the interest rate on money deposits r_M, therefore higher interest rates require higher consumption out of wealth. One can study the dependencies of α_2/r_M on different parameters by calculating the corresponding partial derivative:

$$\frac{\partial \alpha_2/r_M}{\partial \theta} = \frac{\begin{gathered}\omega\lambda(1+\phi)\left(\phi(2-\alpha_1+\alpha_1\theta^2)+\alpha_1(1-\theta)^2 r_L\sigma^\top\right)\\ \dots\ +\phi^2+\omega^2\lambda^2(1+\phi)^2\left(1-\alpha_1+\alpha_1\theta^2\right))\end{gathered}}{-\left[\theta(\phi+\omega\lambda(1+\phi))+\sigma^\top r_L(1-\theta)\right]^2} < 0, \tag{60}$$

$$\frac{\partial \alpha_2/r_M}{\partial \phi} = (1-\theta)^2 \frac{\alpha_1\theta\omega\lambda(1+r_L\sigma^\top)+r_L\sigma^\top\omega\lambda(1-\alpha_1)+r_L\sigma^\top}{\left[\theta(\phi+\omega\lambda(1+\phi))+\sigma^\top r_L(1-\theta)\right]^2} > 0, \tag{61}$$

$$\frac{\partial \alpha_2/r_M}{\partial (r_L\sigma^\top)} = -(1-\theta)^2 \frac{\omega\lambda(1+\phi)(1-\alpha_1(1-\theta))+\phi}{\left[\theta(\phi+\omega\lambda(1+\phi))+\sigma^\top r_L(1-\theta)\right]^2} < 0, \tag{62}$$

$$\frac{\partial \alpha_2/r_M}{\partial \alpha_1} = -\frac{(1-\theta)^2\omega\lambda(1+\phi)}{\theta(\phi+\omega\lambda(1+\phi))+\sigma^\top r_L(1-\theta)} < 0, \tag{63}$$

$$\frac{\partial \alpha_2/r_M}{\partial (\omega\lambda)} = (1-\theta)^2(1+\phi)\frac{r_L\sigma^\top(1-\alpha_1(1-\theta))-\alpha_1\theta\phi}{\left[\theta(\phi+\omega\lambda(1+\phi))+\sigma^\top r_L(1-\theta)\right]^2}. \tag{64}$$

Rising interest payments on deposits, a value proportional to $r_M\sigma^\top$, lower consumption out of wages α_1, lower tax rates θ, and higher markup ϕ increase the minimum value of the fraction α_2/r_M, if a stable stationary economy is desired. The last equation is ambiguous, caused by the fact that ϕ, $\omega\lambda$ and $a_{pe}+a_{ee}$ are dependent via the pricing equation. A higher wage bill at constant profit rate means that intermediate inputs have to decline. Therefore, the sign of the derivative depends on the interaction of increased wage share and the corresponding decrease in intermediate payments.

If no stable fixed point exists, we see an exponential increase of private money deposits and a corresponding growth in public debt, illustrating the accounting principle that all financial assets have symmetrical financial liabilities. Flows of interest payments from the government accumulate and increase the money stock M_h held by the household sector. But if consumption out of wealth α_2 is high enough to counteract the interest and profit payments, households increase their consumption as their stock of wealth increases. The fixed point is stable which enables the economy to remain in stock-flow equilibrium, even though interest rates are positive. It is then not the case that the interest payments drive down government net worth. This shows that the stability of a non-growing economy is indeed a question of the interplay of interest payments

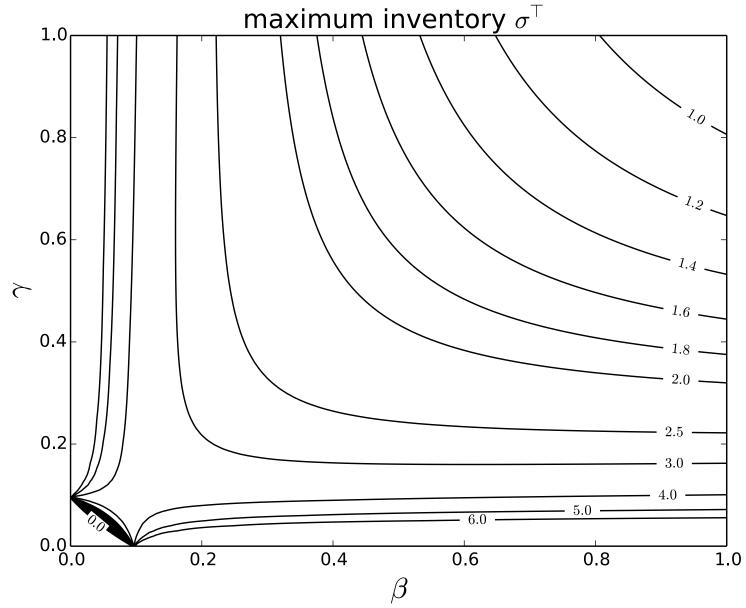

Figure 6: Contour plot of instability induced by inventory oscillations, depicting the maximum possible inventory to expected sales ratio $\sigma^\top$ in order to keep the fixed point stable. In the lower left corner, no stable fixed point exists for any $\sigma^\top \geq 0$.

and the propensity to save, as suggested by Wenzlaff et al. (2014).

For $\alpha_2 = 0.2$, $r_M = 0.05$, $r_L = 0.04$ and $\theta = 0.48$ as in table 3 and for a nominal GDP of $d_pP_p + d_eP_e = 100$, this is realized with $M_h \approx 162.9$, $V_g \approx -86.1$, $L_p \approx 73.7$ and $L_e \approx 3.1$. In this state, the industry sectors realize positive profits ($\Pi_p \approx 45.4$, $\Pi_e \approx 0.7$ per period) which are distributed to the households, the tax income ($T \approx 49.3$) and interest income ($r_LL_g \approx 3.8$) of the government equals the government expenditures ($G \approx 46.6$) and interest costs ($r_MM_h \approx 6.5$), and the total income of the households ($Y \approx 102.7$) equals taxes and consumption ($C_p \approx 51.3$, $C_e \approx 2.1$). Once equilibrium is reached, no sector accumulates any additional stocks, and all income is consumed or distributed, which allows for a stationary economy. Though the model shows that positive interest rates do not necessarily imply exponential growth of liabilities, this result crucially depends on consumption decisions by households.

4.3 Inventory Oscillations

The mapping matrix $\mathcal{M}$ has a pair of complex conjugated eigenvalues for each sector, thus two pairs in the bi-sectoral case. They correspond to inventory cycles (Metzler, 1941). The way the interaction of inventories, expected sales, and realized sales is realized using partial adjustment accelerators leads to a spiral trajectory in the hyperplane spanned by the corresponding eigenvectors. This is caused by the fact that the partial adjustment process includes time lags. If the absolute value of these eigenvalues is smaller than 1, the oscillations of the stock of inventories are damped. As expected and realized sales converge, perturbations return to the fixed point. In the corresponding eigenvector manifolds, this constitutes a stable focus. But if the targeted inventory ratio $\sigma^\top$ is big, a small perturbation in any of the variables may lead to oscillations building up exponentially, thus to an unstable system, described by a complex conjugated pair of eigenvalues with absolute value bigger than 1. The corresponding bifurcation is of Neimark-Sacker type on the center manifolds, though no bounded closed invariant curve is created (Kuznetsov, 2004), but the oscillations diverge.

In figure 6, you can see the maximum of the expected sales to output ratio $\sigma^\top$ allowed in order to keep the equilibrium stable. In the lower left corner, the adjustment parameters are very small, thus adjustment happens very slowly. A small increase in sales that increases production, income, and the monetary stock of household wealth would cause increasing consumption and sales. As the adaptation of enterprises to increasing demand is so slow, they will finally run out of inventories. In this case, no value of $\sigma^\top$ exists which would to keep the system stable.

5 Energy in a SFCIO Model

As explained in section 2.1, energy plays a crucial role in the economic process. We apply our general framework to a model with two goods and sectors: energy and a multi-purpose consumer/industry good. In order to produce the consumer/industry good, the production sector uses energy as well as its own good as inputs, while in order to provide energy, the energy sector uses the Consumer/industry good and the energy good as inputs. This specification ensures that the two sectors are mutually interdependent, and that the model incorporates physical aspects and the dynamics of a monetary production economy. A representation of the flows of money, goods, and energy is given in figure 1 on page 24.

The 'physical quantities' of the IO matrix **a** (20) are defined such that the prices are 1 monetary unit for all goods in the first period, but prices of these quantities may vary over time. The parameters are matched to the situation of Germany around 2010. The IO parameters, the markups, the wage bill, and the consumption vectors are estimated from Statistisches Bundesamt (2010b): For each unit sold, the consumer/industry sector requires an input of 0.48 from its own sector and 0.02 from the energy sector and pays 0.25 units of wages. The energy sector requires 0.60 units from the industry sector and 0.15 units from the energy sector itself and pays 0.13 units of wages. Therefore, the markups on costs can be calculated as $\phi_p = 0.3333$ and $\phi_e = 0.1364$. The tax rate of 0.48 is taken from Statistisches Bundesamt (2010a), the interest rates, accelerators, inventory to sales ratio and consumption parameters are set as rough estimates. All parameters used are displayed in table 3 on page 35.

5.1 Energy Price Shocks

Hamilton (1983, 2013), Murphy et al. (2011), and King (2010) present evidence that every US recession since World War II was accompanied by rising energy prices. It was argued that the income transfers to high saving OPEC countries due to higher oil price are not demand neutral and may have caused the recession (Rubin and Buchanan, 2008).

In Berg et al. (2015b), we studied the impact of a rise in energy markup ϕ_e from $\phi_e = 0.1364$ to 0.4, leading to higher prices. Because the model of a closed economy does not allow to study exports and imports, the higher profit that are not consumed immediately may act as a proxy. The results presented in the paper have one drawback: As government expenditures were kept nominally fixed, in spite of changed prices, the reduction in the equilibrium value was essentially caused by the reduction in real government expenditures following equation (47). The price inflation lead to lower real production, and inversely price deflation would have caused a higher real production. In fact, the fixed point is determined by government expenditures, thus the claims made in Berg et al. (ibid.) concerning the equilibrium value should be judged with caution. Alternatively, the real expenditures may be kept constant. Corresponding plots for both behavioral rules concerning government expenditures are visible for comparison in figure 7.

The increased markup leads to a higher price of energy. In order to incorporate the effect of the low price elasticity, households are assumed to react to an energy price shock by devoting a higher portion of their consumption spending to energy services, so that in the following period, they consume the same amount of energy. This means that C_e^0 changes from 0.039 to 0.048 and remains there. This immediately drives down consumption of other goods $C_p^0{=}1{-}C_e^0$, which reduces the wage in the next period and the expected sales s_p^X of the production sector, leading to a reduction in inventory investment. Additionally, the rise in the price of energy drives up unit costs and therefore also drives up prices in the production sector via the IO interlinkages. Together, this leads to a serious decline in real final demand, which is calculated in prices of the 0^{th} period. However, production goes up again once inventories are reduced, and after the rising profits of the energy sector are distributed to households, households increase their consumption out of wealth. If real expenditures are kept constant, a drastic reduction in the new equilibrium position is not visible, but the maximum temporary reduction in real demand is 2.5%, accounting for the short-term non-neutrality of income transfers to profits.

It is worth noting that the decline of real demand is not easily explained by

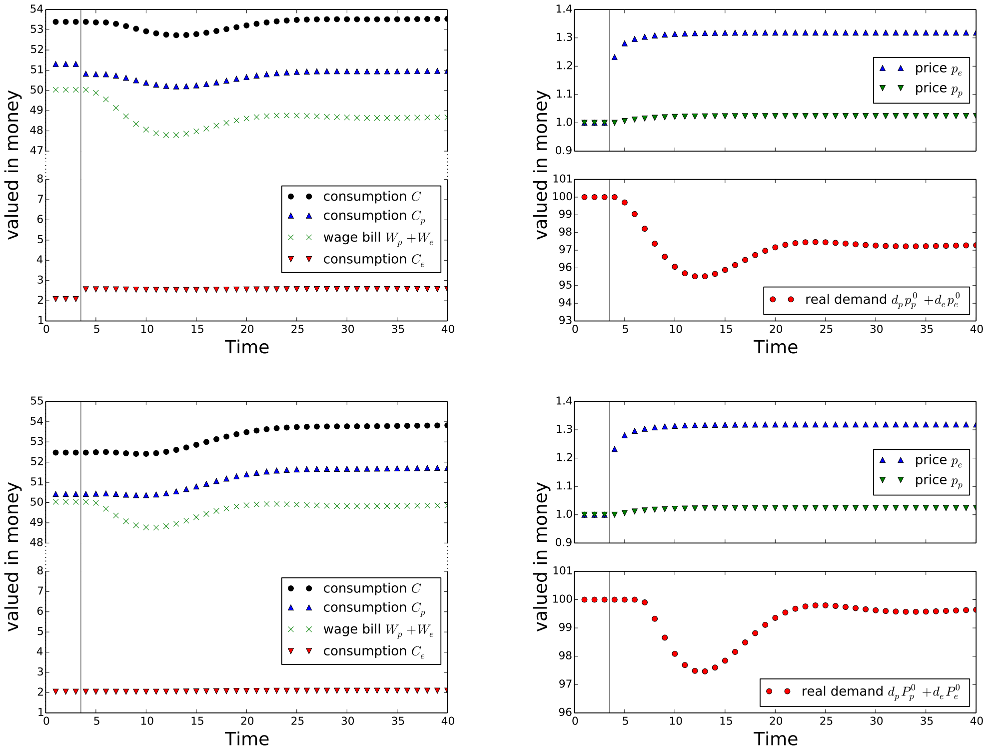

Figure 7: Impact of an increase in energy markup ϕ_e: We initiate the model at the fixed point calculated in section 4.2, and the vertical line indicates the time when the markup on energy is exogenously increased. The left plots shows the decrease of consumption that causes a reduction of demand, which drives down wages and further reduces consumption. The upper two plots keep government expenditures fixed in nominal terms, while the lower plots adapt to price inflation. The right plots indicate the increase in prices caused by the higher energy markup, and the time evolution of real demand, showing a significant temporary decline which is permanent only in the case of nominally fixed government expenditures.

many neoclassical models which abstract from finance and monetary production, in which the reduction of aggregate production would be caused by a reduction of utilization of the energy input, multiplied by the output elasticity. As pointed out in section 2.1, neoclassical authors assumed that output-elasticity should correspond to the cost share, which is around 4% in our model. The reduction of energy consumption by 5% should therefore reduce final demand by 0.2%, which is barely visible. In the simulations, final demand declines temporarily by 2.5%, which is one order of magnitude bigger. Interestingly, this order of magnitude difference has also been realized in the case of historic oil crises (Kümmel, 2011). Within post-Keynesian economics, this impact can be explained by traditional Keynesian multiplier effects (Kemp-Benedict, 2013). So following this interpretation, which is consistent with our model, it was not the reduced supply of oil (no shortage occurs in the model), but the decreased real expenditures that triggered the recession. The drastic increase in energy prices before 2008 may have contributed to the 2007–2008 financial crisis, as reduced growth or lower expected growth may destabilize the economy and the financial system (Tokic, 2012; Wenzlaff et al., 2014).

Multiplier effects play a prominent role in our model and can amplify the negative impact of recessions caused by energy price shocks. An increase in the energy price markup may have the effect of decreasing the real wage, which decreases real consumption (Kemp-Benedict, 2013). First of all, this decline in consumption is amplified by the simple IO output multiplier (Miller et al., 2009, pp. 245–7): If expected final demand is decreased by one unit, total production declines by more than one unit due to intersectoral interlinkages. This multiplier has an immediate effect in our model within each time period. Secondly, a decrease in production leads to a lower labor demand and therefore decreases the wage bill. Households are assumed to immediately spend a fraction of their wages on consumption, and so a lower wage bill decreases consumption. Induced effects of changes in wages which alter consumption of goods across different sectors are referred to in IO models as 'Total Output Multipliers' (ibid., pp. 247–8). Decreased consumption because of lower wages will simply lead to an increase of inventory stocks. This occurs because firms do not decrease their production immediately, since buffering unexpected shocks is the essential role of inventories. In the next period, firms will decrease their expected sales and attempt to reduce their inventory stocks, which will decrease output even more. In principle, all of these propagation and amplification mechanisms can mutually support and strengthen each other, and yield an alternative explanation for the macroeconomic response to energy price shocks that have been observed in the past.

5.2 Energy Returned on Energy Invested (EROI)

Studies have underlined the contemporary significance of energy in terms of the 'Energy Returned on Energy Invested' (EROI), which is the usable energy acquired divided by the amount of energy expended to extract and process that energy resource (Cleveland et al., 1984). It has explanatory power for energy systems as a quantitative means to calculate net flows of energy to the economy for different energy transformation technologies (Kreith et al., 2013, pp. 112ff), providing “a measure of the relative ‘efficiency’ of different energy sources and of the energy system as a whole” (Murphy, 2014). EROI takes into account the energy for manufacture, installation and maintenance of energy transformation systems. While different approaches for calculating EROI exist (Kreith et al., 2013, p. 26), its easiest formulation is a purely physical one as used by Weißbach et al. (2013): The EROI R of a power source is the ratio of usable energy E_R the source returns during its lifetime to all the invested energy E_I needed to make this energy usable:

$$R := \frac{E_R}{E_I}. \tag{65}$$

The standard calculation of EROI includes both indirect and direct energy expenditures of energy extraction by the economy, but costs further downstream, such as transportation and refinement, have been omitted, and more importantly also the fuel extracted or sunlight harvested (Murphy, 2014). Todays average EROI is considered to be between 10 and 20 (ibid.). An EROI below 1 would mean that more energy is consumed than produced by this specific technology, constituting the physical limit.

Declining EROI indicates that the ‘low hanging fruits are picked’ (Kreith et al., 2013, p. 27), and can lead to diminishing availability of energy. As an example, the EROI for conventional oil has declined from over 80 to $15 - 25$ because oil reservoirs are depleted. This explains partly the rising investment costs (ibid., p. 27), and indicates that supplying today’s energy consumption with biodiesel with an EROI around 1.3 (Murphy and Hall, 2010) is extremely hard to achieve, while Biogas (3.0) or photovoltaics (4.0) (Weißbach et al., 2013) are better, but extraction and production is still energy-intensive compared to conventional oil. Fossil fuels provide a very large energy surplus, ‘obviously enough to run and expand the human population and the very large and complex industrial societies around the world’, posing that question about the minimum EROI necessary for society to run, and shifting the focus away from the size of global oil reserves to the question of the size of that portion that is extractable with a positive net energy value and at what rate the high EROI fuels can be produced (Hall et al., 2009).

The input–output model is applied to study the use of energy within the economy in the tradition of Bullard and Herendeen (1975), Carter (1974), and Estrup (1974). The impact of an decrease in EROI is studied by increasing the intermediate inputs required by the energy sector by one third. We keep the other matrix elements fixed and abstract from substitution there, which has been a substantial critique level to IO models (Christ, 1955), but since energy sources such as oil have very low price elasticities (International Energy Agency, 2009; King, 2010) and since we are considering the short term impact of changes in the economy, changes in the other matrix coefficients may be considered small. Therefore, $\boldsymbol{a}$ is adapted accordingly:

$$\mathbf{a} = \begin{bmatrix} 0.48 & 0.60 \\ 0.02 & 0.15 \end{bmatrix} \rightarrow \begin{bmatrix} 0.48 & 0.80 \\ 0.02 & 0.20 \end{bmatrix}. \tag{66}$$

This means that both intermediate inputs of the energy sector were increased by one third, corresponding to a change in EROI. Similar to section 5.1, households increase their relative consumption on energy with rising price. Figure 8 shows the rising prices due to the interlinked sectors, and a decline in energy and products sold to households and the government, while total production in the two sectors increases. The reduction of EROI results in a bigger share of total production used as intermediate products. This numerical result is not surprising at all, because it an expected outcome of increased IO coefficients, and the numerical values are not enlightening, but it shows how declining EROI may be integrated into the model presented.

The approach is subject to critique: Using input–output data to study physical interrelations implies that the output of each sector can be treated as homogeneous, and all intersectoral trade is performed at equal prices, which is unrealistic in the case of the energy sector (Bullard et al., 1975). Using input–output elements as a proxy for EROI is questionable, because IO data contain many costs downstream including distribution to final customers, but this underlines why an EROI of around 3 may be necessary for a sustainable society as suggested by Hall et al. (2009), if operating at EROI $10-20$ (Murphy, 2014) leads to a_{ee}=0.15 which means that not $5-10\%$, but 15% are consumed immediately by the energy sector. Both asks for a more careful data analysis and cross checking with other indicators for energy consumption for future applications (King, 2010).

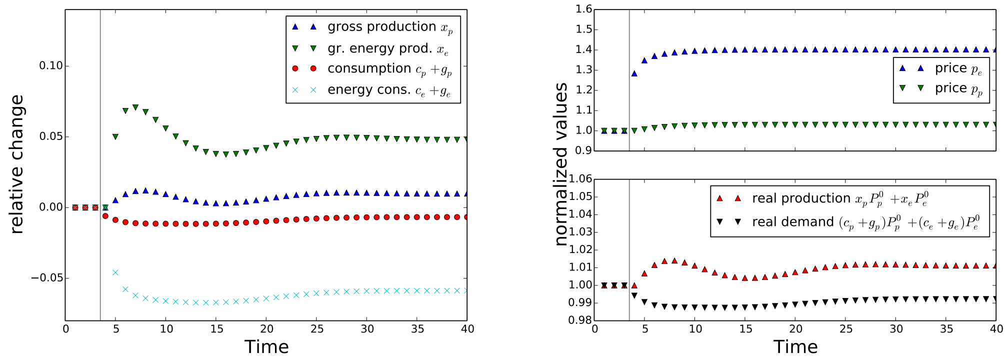

Figure 8: Impact of a decrease in EROI: We initiate the model at the fixed point calculated in section 4.2, and the vertical line indicates the time when the intermediate inputs of the energy sector are increased as a proxy for reduced EROI. The left plots shows the change in production and consumption patterns. The right plots indicate the increase in prices caused by the decline in EROI, and the time evolution of real demand, showing a significant temporary decline, while gross production is increased, indicating that a higher percentage of gross production is used as intermediate input.

5.3 Rebound Effects

William Stanley Jevons (1865) discovered that rising energy efficiency may not lead to a reduction in energy consumption because the improvements may encourage higher-than-otherwise levels of consumption at the economy-wide level (Brookes, 2000, p. 356). The reduction of energy consumption usually falls short of *engineering savings*, the theoretical quantity of energy saved after an increase in energy efficiency if the quantity of goods and services demanded or consumed were held constant. This effect is caused by behavioral or systemic responses known as 'rebound effects', tending to offset a portion of the beneficial effects of the new technology or other measures taken. If the engineering savings are 50%, but the reduction in energy consumption is only 40%, the rebound effect is given by $1-0.4/0.5 = 20\%$ (Madlener et al., 2009; Sorrell and Dimitropoulos, 2008).

We can demonstrate the impact of an increased energy efficiency in our model by cutting by half the input–output parameters a_{ep} and a_{ee}. The *engineering savings* are therefore 50%. One could expect a halving of energy consumption, but the feedback effects in our model lead to an increase in real consumption. The prices in the production sector are reduced by 4.2%, while the energy price is reduced by 12.4%. The decreasing price of goods due to lower energy input

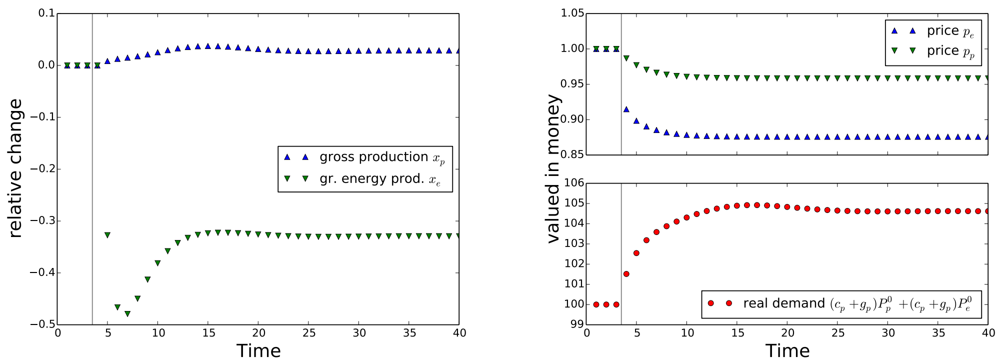

Figure 9: Rebound effect for a doubling of energy efficiency in both the production and the energy sector on their gross production. The left plot shows the relative change in production within the two sectors, showing a decline in the energy sector combined with an increase in the production sector. The right plot indicates the price shift and the time evolution of real demand, valued in prices of the first period.

leads to a *direct* rebound effect: The cheaper prices per unit (visible in figure 9) lead to higher demand: prices in the energy sector are reduced by 12.5%, leading to an increase in consumption of 14.2% keeping energy expenditures fixed. The rest can be attributed to *indirect* and *economy-wide effects*, where the lower price of energy services leads to changes in the demand for other goods, services, and factors of production that also require energy for their provision. They are caused by systemic interlinkages between efficiency changes, prices, income and demand (Madlener et al., 2009; Sorrell et al., 2008).

If we look at the time evolution displayed in figure 9, we see that a temporary reduction in energy consumption close to the engineering savings of 50%, but the economy-wide feedback effects increase consumption subsequently, and the real demand is increased by around 4.6%. The production in the energy sector settles at a decline of 32.9%, while the output of the production sector is increased by 2.9%, as visible in figure 9. The size of the total rebound effect for this improvement in energy efficiency is therefore $\frac{0.5-0.329}{0.5} = 34.2\%$. This corresponds well to the total rebound effect of real economic systems that is estimated to be around $25\% - 40\%$ according to Madlener et al. (2009). Again, the numerical values should only be taken as an indicator of a reasonable behavior of the model economy.

6 A Simple Climate Model with Anthropogenic Heat Emissions

Until now, the model we have considered has been solely an economic model, and although we have depicted material and energy flows crossing the boundaries between the economic system and the ecosystem in figure 1 and discussed the implicit effects of those flows, they were not explicitly incorporated into the model. A broad literature deals with the interconnection between the environment and the economy, in particular the impact of material waste and natural resource scarcity. Emission of heat resulting from thermodynamic principles, however, remains largely neglected, and we conceptualize an integration of heat emissions from economic activities into climate models.

As energy is consumed, the economic process transforms energy into unusable heat (Kümmel, 2011, p. 114). Except for renewable energy sources such as wind, where heat dissipation would have happened anyway, this adds an anthropogenic heat flux whose impact on climate has been discussed e.g. by Chaisson (2008), Döpel (1973), and Washington (1972) or in 'The Limits to Growth' (Meadows et al., 1972, pp. 73f). Today, world average heat emission can be estimated by total primary energy consumption to be around 0.025 Wm^{-2}, which is about 1% of total radiative forcing in 2011 from anthropogenic climate change (Stocker et al., 2013, p. 14). Globally, this may be negligible today (Crutzen, 2004), but is of importance for regional climate models (Flanner, 2009). If energy conversion continues to rise over the course of the century, this may become relevant, especially if new technologies such as nuclear fusion or energy harvesting by satellites (Kümmel, 2011, pp. 76–91) are eventually implemented.

A minimal model can be introduced to get a coarse idea of the impact of human heat flux. We consider a standard model (Bohren and Clothiaux, 2006; Petty, 2006) (see figure 10) in which the Earth is considered as a black body in the infrared spectrum, while the albedo for sunlight is considered to be

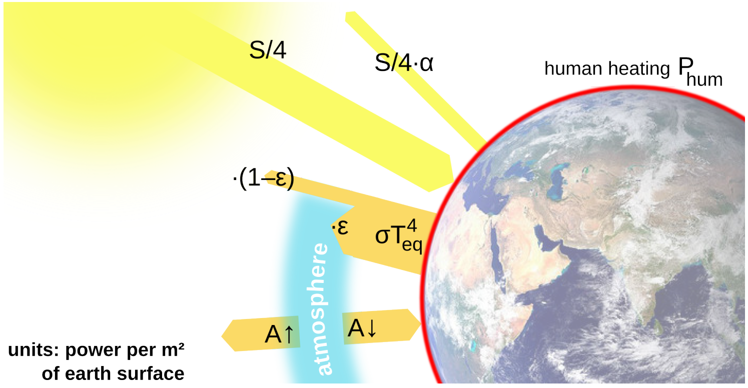

Figure 10: Single layer atmosphere with human heating P_{hum}. The albedo α indicates the fraction of incoming sunlight reflected immediately. At the Earth's crust, a layer of human heat emissions is added. Earth's infrared emissions are due to black body radiation, and a fraction ϵ is absorbed in the atmosphere and radiated evenly in both directions.

$\alpha = 0.3$. The Earth is considered to be at uniform equilibrium temperature T_{eq}. The solar constant is $S = 1370$ Wm^{-2}, leading to a mean insolation of $S/4$, since the surface of a sphere is four times its cross section. The atmosphere is considered as a single layer perfectly transparent for sunlight and with $\epsilon = 0.78$ being the absorptivity and emissivity of the atmosphere in the infrared spectrum. The absorbed radiation is emitted evenly up and down, such that $A\uparrow = A\downarrow = 0.5\epsilon\sigma T_{eq}^4$. As a variation to the standard models, we add a layer of 'human heating' P_{hum} at the Earth's crust. The radiative balance of Earth is given using the Stefan-Boltzmann law by

$$0.25 \cdot S(1-\alpha) + P_{hum} + A\downarrow = \sigma T_{eq}^4, \tag{67}$$

and the equilibrium temperature T_{eq} can be calculated as

$$T_{eq} = \left(\frac{0.25 \cdot S \cdot (1-\alpha) + P_{hum}}{\sigma \cdot (1 - 0.5 \cdot \epsilon)}\right)^{\frac{1}{4}}. \tag{68}$$

For solar energy, this equation has to be adapted slightly: efficient harvesting of sunlight requires low reflection (say: 0), which would lead to an effective albedo of $\alpha_{eff} = \alpha\left(1 - \frac{P_{hum}}{0.25S}\right)$. P_{hum} in the equation must be replaced by αP_{hum} if all thermal power plants were replaced by solar power stations.

Today's energy conversion in Germany accounts for 1.26 Wm^{-2}. If this same degree of energy conversion were to be realized on the whole landmass of the

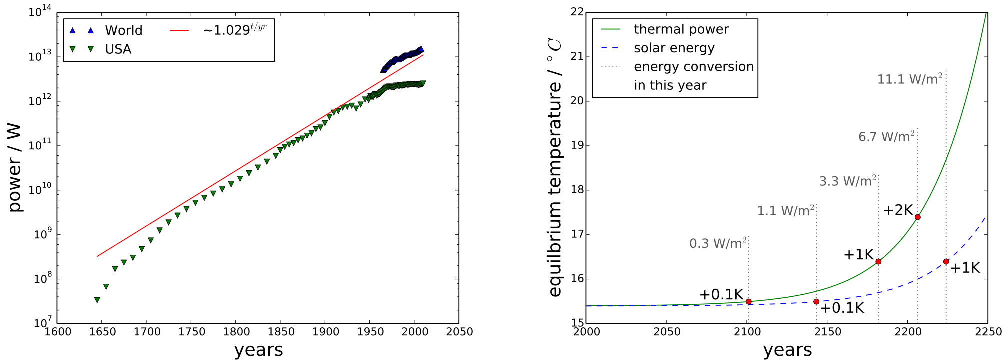

Figure 11: Left: energy conversion in the USA and the world, data from BP (2009) and Energy Information Administration (2012a,b). The solid line indicates an exponentially growing curve with a yearly growth rate of 2.9%. Right: equilibrium temperature of planet Earth calculated by (68), assuming continued exponential growth of energy conversion. The dashed line corresponds to the use of solar energy with the effective albedo α_{eff}.

planet (29.3% of total surface), the temperature increase would be 0.12 K.

In the past, global energy conversion has increased nearly exponentially with a growth rate of around 2.9%, see the left plot in figure 11. If we project this trend into the future, the impact of anthropogenic heat flux could become relevant over the next several centuries, as it would contribute significantly to an increase of average temperature on Earth, see the right plot in figure 11. The temperature rise is smaller for solar energy than for thermal power plants. This demonstrates that the radiation balance of 'Spaceship Earth' (Boulding, 1966) would be significantly affected by a steady increase in energy conversion. A hypothetical continuation of this 2.9% growth rate could break all reasonable limits within centuries, though such an extrapolation would exceed the model's scope. If humans were to discover a cheap, inexhaustible, and environmentally benign source of energy, one might at first glance consider it a clear boon to humanity. However, if its discovery were to lead to increased energy use, heat emissions could potentially have a serious environmental impact. The implementation of new energy technologies could potentially facilitate an explosion of the global population and an increase of consumption, possibly beyond the Earth's sustainable carrying capacity (Kerschner, 2010).

7 Discussion of the Analytical Framework

The development of the stock-flow consistent input–output model in chapter 3 integrated aspects from input–output analysis, post-Keynesian and ecological economics, and constitutes a synthesis of these fields. Several simplification help to keep the model tractable, but from the perspective of the different schools, this causes fundamental drawbacks concerning aggregation, substitution, and scale. Some possible extensions or variations are discussed.

From the perspective of post-Keynesian monetary economics, the complex structure of the banking system, the variety of financial assets and portfolio decisions, but also investment decisions involving long-lived fixed capital assets are missing in the model. But as the model is based on the post-Keynesian SFC approach, the extension to more complex balance sheet and transaction matrices is only a matter of increasing complexity, not of structural or theoretical problems. The integration of investment in fixed capital goods in the model is a necessary condition for developing a post-Keynesian ecological growth model.

From the perspective of ecological economics, the embeddedness of the economic system into the ecosystem and its reliance on energy, resources and space was not included properly into the model. In particular, an explicit treatment of scale is missing (Daly, 1992); the linear structure of the model economy does exactly the opposite. The implementation of a physical scale would have addressed the critique by Daly, and added a non-linearity to the model. This would have lead to a richer dynamical behavior, underlining the need for concepts of dynamical system theory instead of rather simple linear algebra. This linear approach also caused methodological problems, as diverging dynamics had to be treated. If one assumes that economic activity is indeed a dissipative process (Kümmel, 2011), one should think about modeling it as a dissipative dynamical system (Ott, 2002). In the model, the dissipation simply happens in the surrounding ecosystem, but the (arbitrary) extraction of energy is neither restricted nor explicitly modeled. It should be pointed out as well that a stable stationary economy given by a fixed point in monetary terms does not imply an equilibrium state with the environment. These problems were caused by sticking too closely to post-Keynesian demand side reasoning, making it difficult to address supply side constraints consistently.

While it has been underlined that arbitrary substitution is not a realistic assumption because of technological constraints, the constant IO model used here faces the inverse problem by disallowing any adaptive processes within the production sectors. Dynamic input–output (DIO) models that incorporate a feedback effect of investment on future production adjusting the Leontief technical coefficient matrix $\mathbf{a}$ (Miller et al., 2009, pp. 639–42) may help to mitigate this simplification in future work.

Another issue not addressed in the model is aggregation. Though post-Keynesian authors reject the concept of a representative agent, there is not much difference between the assumption of a representative agent and the study of sectoral behavior as in the SFC model presented here. The reliance on a mean field approach excludes heterogeneity, self-organization and emergence (Kirman, 2011, p. 22). The field of complexity economics claims that the economy should be considered as a complex adaptive system, and focuses on interaction, interdependence, networks, trust, and contagion between economic agents. These complex phenomena may cause sudden, endogenously produced changes in system behavior (ibid.), resulting in 'spontaneous emergence of extreme events in self-organizing systems' (Sornette, 2009, p. xv). To relax the assumption of rationality and to consider interaction explicitly, agent-based models (ABM) have been proposed. They can implement locality and search costs, bounded rationality and heterogeneity among consumers, the possibility of coordination failures (Delli Gatti et al., 2011), and defaults and network effects (Battiston et al., 2007). Researches in econophysics have used them to explain distributions with fat tails and volatility clustering (Ballot et al., 2014; Feng et al., 2012). This enables the analysis of emergent disequilibrium dynamics created by the interactions of heterogeneous agents.

As was pioneered by Bergmann (1974), ABMs can also integrate a SFC description of monetary stocks and flows, recently rediscovered including endogenous credit creation (Caiani et al., 2014; Dawid et al., 2012; Kinsella et al., 2011; Riccetti et al., 2014; Seppecher, 2012). Consequently, while the model presented in this paper is not an ABM, it is clear that ABMs offer SFC models a potential method to incorporate a greater degree of heterogeneity. Likewise, the SFC framework offers ABMs a way to implement financial macro constraints, which may help ABMs avert the common criticism that their results are driven too much by the choice of particular parameter values. These innovations may help to transform the SFC perspective from a 'top-down' approach into an agent-based or 'fully-scalable' mode of macroeconomic modeling (Caiani et al., 2014; Dawid et al., 2012). If IO models are also incorporated into the analysis, it would be possible (at least in theory) to trace the implications of the behavior of heterogeneous agents in financial markets on flows of physical materials through the economy as well as through the natural world. Until now, ABM have tended to disregard physical resource flows and energy and therefore miss the 'minimum complexity of endogenous growth models', as claimed by R. U. Ayres (2001).

Human heat emissions were successfully integrated into a simple climate model, but the economic model was only linked implicitly to the climate model. This was unavoidable also because of the structural difference that the climate model is used to estimate the equilibrium temperature of the Earth over long periods of time, whereas the economic model is focused on much shorter-term changes to the structure and size of the economy. This problem may not persist once a more sophisticated climate model is used, capable of studying off-equilibrium dynamics.

The approach offers post-Keynesian economists a possible way to more explicitly incorporate production into their models, and offers ecological economists the opportunity to integrate monetary aspects of the economy into their reasoning. Modelers from econophysicics or agent-based economics may profit from incorporating both production and the symmetry between financial assets and liabilities as an alternative to treating money as a conserved quantity. Though the baseline model proposed here does not capture the rich behavior possible from either approach, the method is designed to enable scaling to an arbitrary number of industries, and also to allow the incorporation of more realistic elements from other already-existing IO models, SFC models, agent-based models, and climate models, that may similarly be studied with methods of dynamical system theory.

8 Conclusion

The thesis conceptualizes the synthesis of disparate insights which have heretofore been developed largely in isolation, in particular post-Keynesian stock-flow consistent (SFC) models, input–output (IO) analysis, and physics (thermodynamics in particular). This is intended to provide an avenue to study the economy and the environment as a unified macroeconomic-ecological system.

A conceptual macroeconomic stock-flow consistent input-output model is presented using mathematical concepts from discrete dynamical system theory. The model consists of a household sector and a consolidated government and banking system sector, along with several industrial sectors.

The stability analysis of the model revealed three instabilities that are all economically meaningful. Studying the price evolution yielded a generalization of the Sraffian maximum rate of profit to a multi-sectoral model with different markups per sector. If markups are high, prices do not converge but diverge, causing the profit share to converge to 1. If markups are below the stability frontier, prices converge to an equilibrium value. The stability frontier corresponding to a transcritical bifurcation could be calculated analytically.

If prices converge to an equilibrium value, the time evolution of the dynamical system can nevertheless be diverging, corresponding to a real eigenvalue of the mapping matrix bigger than 1. The parameter analysis revealed that this instability is caused by the interplay of consumption and interest rates, shedding light on the controversy about whether a non-growing economy is compatible with positive interest rates. The model economy was found to have a stable fixed point if consumption out of wealth is high enough to counteract accumulation. This supports recent claims that the stability of a stationary economy with positive interest depends on consumption decisions. The bifurcation point of the transcritical bifurcation for the corresponding consumption parameter could be derived analytically.

A third instability corresponding to a pair of complex eigenvalues passing the unit circle could be attributed to inventory oscillations. The system undergoes a Neimark-Sacker bifurcation, and the bifurcation point depends on inventory targets and adjustment parameters.

The role of energy use and the energy sector was specifically emphasized as one of the key linkages connecting the natural environment with the economy. The model was applied to some related question relevant for ecological macroeconomics. The impact of energy price shocks on the economy was examined, in particular how rising energy prices can depress real wages, lower demand, and therefore trigger serious recessions. The effect of a decreased Energy Returned on Energy Invested (EROI) on the model economy was studied, showing that this may lead to increased total energy consumption while final demand is reduced. The study of rebound effect yielded an economy-wide effect within the range found by empirical studies. The numerical simulations show that the model can plausibly be applied to such types of problems. As only very few empirical data are investigated, conclusive results could not be expected and cannot be drawn.

To contribute to the study human-nature interaction, the environmental impacts of heat emissions from energy conversion were analyzed. Specifically, implied heat emissions from energy conversion and the effect of anthropogenic heat flux on climate change were considered in light of a minimal single-layer atmosphere climate model. Heat emissions could potentially have a serious environmental impact in the future, if energy use continues to rise. Although this integration of heat into a climate model is very basic, the results underline that heat emissions caused by economic activity should be taken into account in climate modeling once long-term scenarios are examined.

This thesis marks a small step towards conceptualizing a macroeconomic framework which is able to describe a monetary economy within its ecological surroundings. An empirical validation of the model is desirable, but was not performed in this thesis. Aspects from complexity economics and econophysics could additionally be integrated, potentially leading to physical agent-based stock-flow consistent models with explicit treatment of environmental scale, energy use, monetary flows, and interaction effects. This could form a fruitful pluralistic and interdisciplinary research program for different schools of economic thought and the natural sciences. Connecting their insights may lead to a deeper understanding of the economy and help manage a transition towards an environmentally sustainable society.

Appendix: Equation list of Model in section 3

$$V_{h(t)} = M_{h(t)}, \tag{5}$$

$$V_{g(t)} = L_{g(t)} - M_{g(t)}, \tag{6}$$

$$M_{h(t)} = M_{g(t)}, \tag{7}$$

$$L_{g(t)} = \sum_i L_{i(t)}, \tag{8}$$

$$L_{i(t)} = \Psi_{i(t)}, \qquad \Delta L_i = L_{i(t)} - L_{i(t-1)} \tag{9}$$

$$V_{h(t)} + V_{g(t)} = \sum_i \Psi_{i(t)}, \tag{10}$$

$$C = \alpha_1(1-\theta)\text{Ш} + \alpha_2 M_{h(t-1)}, \tag{11}$$

$$C_i = CC_i^0 \leq C \qquad \text{with} \sum_j C_j^0 = 1, \tag{12}$$

$$c_i = C_i/P_i \qquad \forall\ i, \tag{13}$$

$$Y = \sum_i \text{Ш}_i + \sum_i \Pi_i + r_M M_{h(t-1)}, \tag{14}$$

$$M_{h(t)} = (1+r_M)M_{h(t-1)} + \sum_i \left(\text{Ш}_i + \Pi_i - C_i\right) - T, \tag{15}$$

$$T = \theta \cdot Y, \tag{16}$$

$$g_i = G_i/P_i \qquad \forall\ i, \tag{17}$$

$$M_{g(t)} = (1+r_M)M_{g(t-1)} + \sum_i \left(G_i + \Delta L_i - r_L L_i\right) - T, \tag{18}$$

$$\mathbf{a} = (a_{ij}), \tag{19}$$

$$\mathbf{P} = \mathbf{diag}\,(P_i) \qquad \Leftrightarrow \qquad P_{ij} = P_i \delta_{ij}, \tag{20}$$

$$\mathbf{A} = \mathbf{P} \cdot \mathbf{a} \qquad \Leftrightarrow \qquad A_{ij} = a_{ij} P_i, \tag{21}$$

$$\boldsymbol{s_{(t)}} = \boldsymbol{c} + \boldsymbol{\xi} + \boldsymbol{g}, \tag{22}$$

$$\boldsymbol{s^X_{(t)}} = \beta \boldsymbol{s_{(t-1)}} + (1-\beta)\boldsymbol{s^X_{(t-1)}}, \tag{23}$$

$$\boldsymbol{\psi}^\top = \sigma^\top \boldsymbol{s^X_{(t)}}, \tag{24}$$

$$\boldsymbol{\Delta\psi}^\top = \gamma\left[\boldsymbol{\psi}^\top - \boldsymbol{\psi_{(t-1)}}\right], \tag{25}$$

$$\boldsymbol{x} = \boldsymbol{s^X_{(t)}} + \boldsymbol{\Delta\psi}^\top, \tag{26}$$

$$l_i = \lambda_i x_i, \tag{27}$$

$$\text{Ш} = \sum_i \text{Ш}_i = \sum_i \omega_i \lambda_i x_i, \tag{28}$$

$$\boldsymbol{d} = (\mathbf{1} - \mathbf{a})\,\boldsymbol{x}, \tag{29}$$

$$\boldsymbol{\xi} = \mathbf{a} \cdot \boldsymbol{x}, \tag{30}$$

$$P_{i(t)} = \left(1 + \phi_{i(t)}\right)\left[\omega_{i(t-1)}\lambda_{i(t-1)} + \sum_k P_{k(t-1)} a_{ki(t-1)}\right], \tag{31}$$

$$\mathbf{E} = \mathbf{P}\ \mathbf{a}\ \mathbf{diag}(x_i) \qquad \Leftrightarrow \qquad E_{ij} = a_{ij} P_i x_j, \tag{32}$$

$$\boldsymbol{\psi_{(t)}} = \boldsymbol{\psi_{(t-1)}} + \boldsymbol{x_{(t)}} - \boldsymbol{s_{(t)}}, \tag{33}$$

$$L_{i(t)} = \Psi_{i(t)} = \psi_{i(t)}\left[\omega_{i(t-1)}\lambda_{i(t-1)} + \sum_k P_{k(t-1)} a_{ki(t-1)}\right], \tag{34}$$

$$\Pi_i = C_i + G_i - \text{Ш}_i + \sum_j E_{ij} - \sum_j E_{ji} - r_L L_{i(t-1)} + \Delta\Psi_i. \tag{35}$$

All matrices are displayed as bold roman letters, vectors in bold italic characters. $\mathbf{diag}(x_i)$ indicates a diagonal matrix with x_i on the diagonal.

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Analytical solutions were obtained using significant amounts of pen, paper, and eraser (Munroe, 2007). Numerical calculations and plots were performed using LibreOffice 4.2.8.2, Python 2.7.6, scipy 0.13.3, numpy 1.8.2, and matplotlib 1.3.1.

Schriftliche Erklärung

Hiermit versichere ich, dass ich diese Arbeit selbständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe. Außerdem versichere ich, dass ich die allgemeinen Prinzipien wissenschaftlicher Arbeit und Veröffentlichung, wie sie in den Leitlinien guter wissenschaftlicher Praxis der Carl von Ossietzky Universität Oldenburg festgelegt sind, befolgt habe.

Oldenburg, den 19. Juni 2015