Between bounded rationality and economic imperatives: essays on out-of-equilibrium dynamics

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1 Synopsis

> "Imagine how hard physics would be if electrons had feelings."
>
> – attributed to Richard P. Feynman –

1.1 Introduction

The five self-contained research papers of this dissertation address two topics: I discuss the economic imperatives and political constraints hindering the economic transformation towards sustainability, and introduce a novel economic out-of-equilibrium framework to model the interaction of bounded rationality and economic constraints without making prior equilibrium assumptions. Both subjects are part of a long standing debate in the social sciences, namely how structure and agency influence human behavior.

The first two papers (with Andreas Siemoneit) contribute to the debate on 'green growth' vs. 'economies beyond growth' as means to substantially reduce resource consumption and environmental impact and to stay within 'planetary boundaries'. Chapter 2 discusses whether growth policies are only a question of political or individual will, or whether economic 'growth imperatives' make them inescapable. We derive micro level definitions of 'social coercion' and 'growth imperative' and structure the debate, differentiating behavioral motives from free will to social influence to social coercion. For firms, households, and nation states, we identify the economic conditions, collective convictions, and political constraints that make alternatives to fostering economic growth 'unrealistic'. Chapter 3 clarifies why certain modeling approaches lead to a 'monetary growth imperative', such that the structure of the monetary system destabilizes a non-growing economy. By introducing the concept of stability analysis to post-Keynesian Stock-Flow Consistent (SFC) models, we show that if the stationary state is unstable, it is caused by agents' decisions, not by structural inevitableness.

Chapters 4, 5, and 6 present the General Constrained Dynamics (GCD) framework to model the interaction of bounded rationality and economic constraints. Chapter 4 (with Erhard and Florentin Glötzl) presents the out-of-equilibrium framework and a dynamic Edgeworth box

exchange model with slow price adaptation. Chapter 5 (with Erhard Glötzl) shows how to transform an existing post-Keynesian SFC model into a GCD model by substituting behavioral functions by the gradient climbing of a utility function that depends on consumption and holding of liquidity. Chapter 6 describes a complex production model based on a Keynesian balance sheet approach and studies the conditions for convergence to a neoclassical equilibrium. The framework can integrate aspects of different economic modeling approaches, from general equilibrium models to Keynesian disequilibrium to agent-based models.

1.2 Background

The debate on sustainable development is controversial between supporters of "green growth" as opposed to those who stress the "limits of growth". The former suggest to decouple environmental impact and economic activity (Jacobs, 2013), an idea already at the heart of the Brundtland report (World Commission on Environment and Development, 1987) or the Stern report (Stern, 2006): Stern argues that the costs of preventing environmental damage are relatively low such that economic growth was still possible, and the threat of losses in future growth rates would be greater if the environment was not protected. The concept of green growth continuously gained attention (Jacobs, 2013) and is at the core of political strategies of (OECD, 2011, 2015) and World Bank (2012) to achieve the goal of sustainable development by promoting innovation and investment. In comparison, concepts such as the "Green Economy" by UNEP (2011b) or eco-social market economy (Jackson, 2009) put forward a broader perspective on well-being and social justice, and the social movement of "degrowth" calls for a radical political and economic reorganization (Kallis et al., 2018).

On the one hand, empirical studies question the sustainability of the green growth strategy, as actual reductions of environmental impact have been rare and of insufficient magnitude in the past (UNEP, 2011a; Ward et al., 2016; Akizu-Gardoki et al., 2018; International Resource Panel, 2019; Parrique et al., 2019). On the other hand, there is general agreement among the representatives of green growth that the necessary environmental protection cannot be achieved without improved policies that have not yet been implemented (Jacobs, 2013). Critics of green growth conversely fear that necessary sustainable policies may not be implemented if they come into conflict with the prominent political goal of economic growth (Jackson, 2009; Kallis et al., 2018).

But are growth policies only a question of political or individual will? Or do mechanisms exist that require economic growth to keep the economy functioning and are hard to circumvent for individuals, firms, or nation states? A 'growth imperative' could make a departure from growth policies within the current economic order impossible. The debate goes as far back to Marx (1906 [1867], p. 649) describing the capitalist as subject to a growth imperative,

because competition creates "external coercive laws" and "compels him to keep constantly extending his capital [. . .] by progressive accumulation". Since then this thesis has been highly controversial, but mostly disregarded in neoclassical and Keynesian growth theories (Lange, 2018; Binswanger, 2019). A stringent review of the topic based on precise definitions is lacking, and chapters 2 and 3 attempt to fill this gap.

The discussion on growth imperatives is a tiny part of a long standing controversy in the social sciences, discussing the dichotomy of 'structure and agency' to explain human behavior: Structure refers to institutions or economic conditions that limit the choices and opportunities available, while agency captures the ability of individuals to make decisions. While the structure–agency debate has been a controversial issue in sociology (O'Donnell, 2010), in economics the distinction is usually very clear: Based on individual rationality, consumers take deliberate decisions based on their 'eccentric' desires (Lancaster, 1971, p. 23), but constraints arising from the market structure of the economy limit their choices: Dixit (1990, p. 1) even identifies the concept of maximizing utility functions subject to market constraints as a principle that defines the science of economics.

The schools of economic thought differ in their ways of modeling the interplay of agency and the economic constraints. In general, a dynamic economic model is described by i variables that can correspond to any stocks or flows of commodities, resources, financial liabilities, prices, or parameters. The *structure* of the model consists of k economic constraints that remove many degrees of freedom. The respect for identities is "the beginning of wisdom" in economics, but they must not be "misused to imply causation" (Tobin, 1995, p. 11). To derive causal arguments, a 'closure' has to be chosen that combines individual agency and the constraints: If the i variables were influenced by i behavioral equations, the system of equations would be overdetermined because of the additional k constraints (Taylor, 1991).

In general equilibrium models, each agent fully controls and voluntarily adapts all the stocks and flows directly affecting him. Price adaptation leads to a market equilibrium that makes all the individual plans compatible with each other (Arrow and Hahn, 1971; Mas-Colell et al., 1995). Introducing bounded rationality, heterogeneity in preferences or social interactions may, however, have the consequence that no unique and stable equilibrium can be guaranteed (Stoker, 1993; Kirman, 2010).

Keynesian disequilibrium models depart from the assumption that price adaptations equalize the quantities of demand and supply. This results in "false trading" at prices that are not market-clearing: Exchange is either rationed by the 'short-side', thus the minimum value of supply and demand (Benassy, 1975; Malinvaud, 1977), or purely demand-led as the labor market in some post-Keynesian Stock-Flow Consistent (SFC) models (Godley and Lavoie, 2012; Caverzasi and Godin, 2015). Terms such as "forced saving" or "involuntary unemployment" (Barro and Grossman, 1971) reflect the fact that agents cannot have complete control over the

variables affecting them. The *ex-ante* (planned) behavior and the *ex-post* (actual) dynamics differ.

Agent-based models (ABM) describe the interactions of heterogeneous and bounded rational agents. They implement insights from psychology, sociology and behavioral economics, emphasizing the role of institutions, power relations, social norms and interaction in influencing economic choices (Akerlof, 2002; Harstad and Selten, 2013). ABM lack a common core, and different coordinating mechanisms such as price adaptation, auctions, matching algorithms, or quantity rationing are implemented (Tesfatsion, 2006; Gintis, 2007; Gallegati and Richiardi, 2009; Ballot et al., 2014; Riccetti et al., 2015). Unfortunately, many ABM fail to satisfy stock-flow consistency: For example, in the exit-entry process of firms, defaulted firms are often recapitalized "ex-nihilo", violating economic identities and leading to a logically incoherent evolution of stocks and flows. This led to calls for stock-flow consistent agent-based modeling (Caiani et al., 2016; Caverzasi and Russo, 2018).

Despite the differences, general equilibrium, Keynesian disequilibrium and agent-based models share the common challenge to describe the time evolution of stocks, flows and other variables subject to economic constraints. This dissertation attempts to bridge the methodological gaps between those approaches by conceptualizing a dynamic modeling framework that describes the interaction of bounded rational agents subject to economic constraints. The simultaneous processes of trade and price adaptation may dynamically converge towards an equilibrium, but no prior assumptions about the nature of market equilibria are made. Nevertheless, the model should include well-known solutions of general equilibrium models as fixed points of the dynamical system in order to integrate different economic theories within a joint mathematical framework.

Historically, static equilibrium theory as introduced by the early neoclassicals such as Vilfredo Pareto, Irving Fisher or Léon Walras was inspired by Newtonian physics (Pikler, 1955; Mirowski, 1989; Leijonhufvud, 2006). But in classical mechanics, the formalism introduced by Lagrange (1788) can describe *motion* under constraint (Flannery, 2011). The idea of this dissertation is to extend the historical analogies from constrained optimization to constrained dynamics, such that the adaptive economic *processes* can be modeled, not only the equilibrium states. To describe the dynamics of bounded rationality and economic imperatives, economic (constraint) forces and economic power have to be formalized in analogy to physical (constraint) forces and the reciprocal value of mass. Chapters 4–6 introduce the General Constrained Dynamics framework and apply it to an exchange model, a post-Keynesian Stock-Flow Consistent model and a complex production model. The assumptions about structure and agency from different schools of economic thought can be integrated into a joint modeling framework.

1.3 Research Contributions and Status of the Papers

Table 1 provides an overview of the articles' titles, co-authorship and publication status. Four out of five papers have been published in peer-reviewed journals, the last has been submitted. The main contributions of chapters 2 to 6 of this dissertation are summarized in the following.

Table 1: Articles' titles, co-authorship and submission status of the five papers

Chapter 2	*Growth imperatives: Substantiating a contested concept.*
	by Oliver Richters and Andreas Siemoneit
	published in: Structural Change and Economic Dynamics 51, December 2019, pp. 126–137. doi:10.1016/j.strueco.2019.07.012.
Chapter 3	*Consistency and stability analysis of models of a monetary growth imperative.*
	by Oliver Richters and Andreas Siemoneit
	published in: Ecological Economics 126, June 2017, pp. 114–125. doi:10.1016/j.ecolecon.2017.01.017.
Chapter 4	*From constrained optimization to constrained dynamics: extending analogies between economics and mechanics.*
	by Erhard Glötzl, Florentin Glötzl and Oliver Richters
	published in: Journal of Economic Interaction and Coordination 14(3), September 2019, pp. 623–642. doi:10.1007/s11403-019-00252-7.
Chapter 5	*Modeling economic forces, power relations, and stock-flow consistency: a general constrained dynamics approach.*
	by Oliver Richters and Erhard Glötzl
	published in: Journal of Post Keynesian Economics 43(2), 2020, pp. 281–297. doi:10.1080/01603477.2020.1713008.
Chapter 6	*Modeling the out-of-equilibrium dynamics of bounded rationality and economic constraints.*
	by Oliver Richters
	submitted to: Journal of Economic Behavior & Organization, preprint published as Oldenburg Discussion Paper in Economics V-429-20. www.uol.de/fileadmin/user_upload/wire/fachgebiete/vwl/V-429-20.pdf.

Growth imperatives: Substantiating a contested concept

Chapter 2 (with Andreas Siemoneit, *Struct Change Econ Dyn* 2019) discusses whether economic growth policies are 'only' a question of political or individual will, or whether 'growth imperatives' make them inescapable. This debate has been controversial, but our literature

review revealed that some of the arguments were disputed simply because of the different use of terms. For example, in the debate on "profits", no clear distinction was made between accounting profit and economic profit. Therefore, we could refute arguments claiming that the mechanisms of a market economy as such create a growth imperative. As a first contribution, we derive micro level definitions of 'social coercion' and 'growth imperative' that were previously missing. It is a core aim of the paper to demarcate the terms 'social coercion' from 'social pressure' as well as 'growth imperative' from 'growth driver', and to explain how they relate to each other. We consider the terms 'coercion' and 'imperative' appropriate only if the agent has to choose a certain alternative in order to avoid existential consequences such as loss of income. For example, if the "perennial gale of creative destruction" strikes at the "very lives" of firms (Schumpeter, 1942, p. 84), the impending loss of income can have existential consequences and 'force' certain company decisions.

The second contribution of the paper is to scrutinize the motives for economic growth discussed in economic and sociological literature. We structure the debate along two dimensions: (a) degree of coerciveness from free will to social conformity to social pressure to social coercion, and (b) agents affected. We analyze different theories for firms, households, and nation states and come to the following conclusions: Firms are subject to a growth imperative because entrepreneurs can reduce costs and prices or improve product value with relatively cheap factor combinations of capital and energy that substitute human labor. This competitive advantage establishes a general trend toward process automation and a bias for technical products. For households, technical consumption goods that make private life more efficient become existential necessities. The availability of natural resources plays a key role in trapping firms and households (supply and demand) in a positive feedback loop. Concerning nation states, we describe how the growth imperatives of the economic agents translate into a political growth imperative, because certain collective convictions and political restrictions make alternatives to fostering economic growth 'unrealistic': the combination of (1) resource-intensive technology that redistributes income and raises labor productivity, (2) the social necessity of a minimal standard of living and (3) the meritocratic principle as a guiding social and political norm which limits the political possibilities for redistribution. The resulting dilemma between 'technological unemployment' and the social necessity of high employment explains why states 'must' foster economic growth.

The theory we have developed has the advantage of cultural and normative parsimony. It sheds light on a number of social riddles, e.g., the attractiveness of technology, the possibility to extort society with the potential losses of jobs or the historically perceived asymmetry of power between employers and employees. The economic attractiveness of resource use makes it necessary to regulate resource consumption politically in order to stay within planetary boundaries. Proposals for market-compliant regulations of resource consumption should be

combined with a redistribution of the revenues to households (cap, tax, or fee and dividend). This would push innovations into a new direction probably less threatening for the environment while reducing inequality and the political need for economic growth. Implementing institutions that limit resource consumption and prevent or redistribute resource rents is more than just a measure for more ecological sustainability. If the meritocratic principle is a foundational social norm, then tackling economic rents such as land rents could provide a general political compass toward a just and sustainable society based on market economy.

Consistency and stability analysis of models of a monetary growth imperative

Chapter 3 (with Andreas Siemoneit, *Ecol Econ* 2017) examines theories of a 'monetary growth imperative'. Several authors argued that the monetary system destabilizes a non-growing economy, which would imply a very strong non-neutrality of money. The thesis received some attention in the degrowth movement (Kallis et al., 2018) and in the German-speaking political and scientific debate (Strunz et al., 2017; Mugier, 2019), for example in the enquete commission on 'growth, prosperity and quality of life' by the German parliament (Deutscher Bundestag, 2013). The central aim of this paper is to clarify why certain modeling approaches lead to a growth imperative and others do not. All investigated discrete time out-of-equilibrium models follow the post-Keynesian tradition of monetary circuit theory, essentially trying to answer these questions: "Where is all the money going? Where do all the money come from?" (Chesnutt, 2012). Our paper makes two contributions: First, it introduces the concept of stability analysis known from dynamic system theory to Stock-Flow Consistent (SFC) models. Second, we provide such a systematic analysis of the stability conditions of several models and analyze their economic consistency.

On the one hand, the paper discusses the claims by Beltrani (1999), M. Binswanger (2009b) and H. C. Binswanger (2009a, 2013) that a non-growing economy was unstable independent of the will of the economic agents, because positive interest rates would lead to negative business profits. First, we reveal a discrepancy between text and model: the authors write that bank's equity capital has to increase with credit volume, but model it such that it steadily increases *even if debt does not.* Second, we show that the growth imperative arises from inconsistent modeling. Therefore, we consider these theories of a growth imperative to be refutable. In his new book, M. Binswanger (2019) eradicates the accounting error with reference to our paper, and the modified model now largely confirms our analysis.

On the other hand, we analyze five post-Keynesian Stock-Flow Consistent (SFC) models. SFC models are a class of structural macroeconomic models based on a detailed and careful articulation of balance sheets and accounting relationships (Caverzasi and Godin, 2015). Godley and Lavoie (2012), Jackson and Victor (2015) and Cahen-Fourot and Lavoie (2016) could

not identify any growth imperative in their models, but were lacking a formal derivation of the stability conditions. We differentiate their claim, showing that it depends on the consumption parameters whether a stationary state can be reached, similar to the results by Berg et al. (2015). High interest rates with low consumption out of wealth can render the model unstable and lead to limitless debt accumulation, as a no-ponzi condition is missing in these models.

We conclude that a monetary system based on interest-bearing credit money with private banks does not lead to an 'inherent' growth imperative. If the stationary state is unstable, it is caused by agents' saving and investment decisions, not by structural inevitableness. A theory of a growth imperative has to study the incentives and constraints of individual agency, instead of just relying on structuralist arguments.

From constrained optimization to constrained dynamics: extending analogies between economics and mechanics

Chapter 4 (with Erhard Glötzl and Florentin Glötzl, *J Econ Interact Coord* 2019) introduces a modeling framework called "General Constrained Dynamics" (GCD). Historically, economic equilibrium models have been inspired by analogies to stationary states in classical mechanics, and we extend these mathematical analogies from constrained optimization to constrained dynamics. The idea is to model the various (market) forces within the economy in a consistent way, based on the concept of constrained dynamics from Lagrangian mechanics (Lagrange, 1788; Flannery, 2011) Similar to the forces of interacting 'bodies' under constraint from Lagrangian mechanics, the modeling approach depicts the economy from the perspective of economic forces and economic power. Economic force corresponds to the desire of agents to change certain variables, while economic power captures their ability to assert their interest to change them. Optimization is replaced by a gradient seeking approach in line with procedural rationality discussed in behavioral economics. These *ex-ante* forces are completed by constraint forces from unanticipated system constraints to yield the *ex-post* dynamics.

The differential-algebraic equation framework overcomes some restrictions inherent to optimization approaches of conventional economic models, namely presupposing that individual utility functions can to be aggregated into a social welfare function (the aggregation problem), and the presupposition of market equilibria. In this framework, agents are unable to calculate infinite dimensional intertemporal optimization problems based on rational expectations about the reactions of the other market participants. Instead, they base their decisions on how much to work, invest, consume, or save on the observation of current marginal utilities, profits, productivities, and prices. They do not jump to the point of highest utility as rational utility maximizers, but instead try to 'climb up the utility hill' gradually by pushing the economy in the direction of highest marginal utility. In a continuous time framework, this can be modeled

by defining the forces exerted by the agents as gradients of their utility functions. The out-of-equilibrium framework describes the interaction of these bounded rational agents trying to improve their situation subject to the economic constraints. Trade continuously takes place at current prices, that react slowly to imbalances between demand and supply.

As an example, we present a simple exchange Edgeworth box economy with two agents and two goods. Instead of the usual static optimization solution, we provide a dynamic model with gradient climbing and slow price adaptation. The relevant system constraints are the budget equation and the restrictions of exchange. Analytically and numerically, we study the time evolution that may dynamically converge towards equilibrium. If the auctioneer knows the price and has perfect control, we obtain convergence to the 'standard' solution. If the 'economic power' of the auctioneer is positive but finite, the sets of rest points of the static and the dynamic model are identical, but the final allocation diverges from the standard equilibrium value.

Modeling economic forces, power relations, and stock-flow consistency: a general constrained dynamics approach

Chapter 5 (with Erhard Glötzl, *J Post Keynes Econ* 2020) applies the GCD framework to a simple monetary Stock-Flow Consistent (SFC) model in the post-Keynesian tradition. We transform a simple SFC model from the textbook by Godley and Lavoie (2012) and reveal its implicit assumptions about power relations and agents' preferences. The system constraints are the balance sheets and budget constraints of the individual sectors. We show how the behavioral equations used in post-Keynesian SFC modeling can be understood as agents trying to improve their utility that depends on consumption and holding of liquidity.

The GCD framework has several advantages over conventional SFC models: First, a behavioral influence can be assigned to each of the variables, instead of just dropping one behavioral equation for each constraint to avoid an overdetermined system. Second, mixed power relations with joint influences depending on the power relations can be directly modeled. As "the distribution of income and power is a basic concern of Post Keynesians" (Davidson, 1980, p. 162, see also Lawson, 1994; King, 1996), the GCD modeling concept may prove beneficial for this school of thought. Third, the preferences of agents and the constraints can be expressed similarly to general equilibrium model, facilitating a comparison and integration of different modeling approaches.

Modeling the out-of-equilibrium dynamics of bounded rationality and economic constraints

Chapter 6 (2020) provides a complex macroeconomic model to show how ideas from general equilibrium, Keynesian disequilibrium and agent-based models can be integrated into a joint mathematical framework. The model contains two households, two production sectors with input–output relations, banks, and the government. The model is based on a Keynesian balance sheet approach, where money is created endogenously and prices react slowly on supply–demand mismatches. Firms are endowed with capital and inventories, financed by equity and credit. All agents follow a gradient climbing approach and gradually adjust quantities.

Analytical calculation reveals that the stationary states of the model satisfy all the conditions usually assumed in static neoclassical general equilibrium models, but the stationary state is path-dependent on the out-of-equilibrium dynamics. The numerical analysis of different adaptation speeds for quantities and prices shows that if quantity adjustments are fast, they can amplify deviations from the equilibrium. The reason is that the bounded rational firms do not correctly anticipate the reactions of the other market participants to their change in production. Instead, they react on supply–demand mismatches by adapting production and prices. Different from DSGE models, fast adaptation of quantities and prices does not lead to fast convergence, but the existence of frictions has a stabilizing effect.

1.4 Conclusion

Contributing to the debate on sustainable development, the papers on economic growth imperatives showed that simplistic arguments based on structuralist theories can be refuted, but a convincing theory has to study the incentives and constraints of individual agency and policy making. According to our analysis, the 'growth imperative' that hinders the economic transformation towards sustainability arises out of the combination of three factors: First, resource-intensive technology redistributes income and raises labor productivity. Second, each members of society should be provided with a minimal standard of living. Third, the meritocratic principle limits the political possibilities for redistribution.

We suggest to use this third factor as a political compass for a just and sustainable market economy. Systematic deviations from the meritocratic principle are a good starting point for policy interventions that combine ecological sustainability, economic efficiency and social justice instead of playing them off against each other. Ecological degradation and social injustice are not necessary consequences of market economies, as claimed in the literature on 'degrowth' (Kallis et al., 2018), but rather the result of missing political limits on emissions and raw material consumption. Correcting externalities reduces market inefficiencies, but also implements the principle of meritocratic justice: those that have the benefit should also bear the cost.

In the papers, we addressed the debate on growth imperatives from a purely theoretical perspective with reference to empirical analyses. In principle, the General Constrained Dynamics framework allows to model the identified individual and political constraints on decision making, such as institutions, social norms and economic imperatives. To model the dynamics of innovations discussed in chapter 2 would require the integration of concepts from endogenous growth theory into the framework. Further extensions of the model could include a more realistic treatment of banks and the financial sector, but also of the ecosystem whose dynamics and tipping points are influenced by economic activity. Emissions along with material and energy extraction could be integrated into a more sophisticated production theory. Further 'social pressures' or 'growth drivers' from sociological theories could be implemented as interactions between households.

This dissertation can be only a first step towards an empirically founded use of the General Constrained Dynamic framework, but the papers demonstrate the wide range of possibilities it entails. The stability analysis revealed that reducing frictions and increasing adaptation speeds may be destabilizing, because the agents do not anticipate the reactions of other market participants to their actions. It remains unsettled whether this result holds if forward looking expectations of firms and the related intertemporal coordination problem are integrated. If this was the case, political regulation should concentrate on designing market frictions to stabilize markets, instead of eliminating them.

Implementing the General Constrained Dynamic models, production and utility functions were chosen such that the dynamics converge to stable equilibria for most parameters. Economic models with multiple equilibria typically incorporate incomplete markets due to transaction costs or information asymmetries, increasing returns to scale, or market imperfections such as entry costs or external effects (Benhabib and Farmer, 1999). They were studied to explain issues such as asset bubbles, collateral shortages, liquidity dry-ups, bank runs, or financial crises (Miao, 2016). If multiple equilibria exist, a theory that describes the out-of-equilibrium dynamics is required to determine which of the equilibrium states is reached. A drawback of the GCD approach is that general equilibrium models with multiple markets are tremendously complex in the amount of variables that are simultaneously 'in equilibrium'. Consequently, providing models able to describe genuine out-of-equilibrium dynamics for all these variables poses a significant challenge. An intermediate approach could combine equilibrium dynamics with out-of-equilibrium processes where necessary. As the concept of Lagrangian closure draws on a mathematical similarity to static optimization models, the General Constrained Dynamics framework is a suitable candidate for this task.

Overall, the author has to state that this dissertation project has raised more questions than it has provided answers. Studying the out-of-equilibrium dynamics of bounded rationality and economic imperatives remains a challenge.

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2 Growth imperatives: Substantiating a contested concept

Authors: Oliver Richters and Andreas Siemoneit

Abstract: Economic growth remains a prominent political goal, despite its conflicts with ecological sustainability. Are growth policies only a question of political or individual will, or do 'growth imperatives' make them inescapable? We structure the debate along two dimensions: (a) degree of coerciveness between free will and coercion, and (b) agents affected. With carefully derived micro level definitions of 'social coercion' and 'growth imperative', we discuss several mechanisms suspected to make growth necessary for firms, households, and nation states. We identify technological innovations as a systematic necessity to net invest, trapping firms and households in a positive feedback loop to increase efficiency. Resource-intensive technology is economically attractive because of a subtle violation of the meritocratic principle of justice. The resulting dilemma between 'technological unemployment' and the social necessity of high employment explains why states 'must' foster economic growth. Politically, we suggest to institutionally limit resource consumption and redistribute economic rents.

3 Consistency and stability analysis of models of a monetary growth imperative

Authors: Oliver Richters and Andreas Siemoneit

Abstract: Is fostering economic growth 'only' a question of political will or 'unavoidable' to maintain economic stability? It is disputed whether such a 'growth imperative' is located within the current monetary system, creating conflicts with sustainability. To examine the claim that compound interest compels economies to grow, we present five post-Keynesian models and show how to perform a stability analysis in the parameter space. A stationary state with zero net saving and investment can be reached with positive interest rates, if the parameter 'consumption out of wealth' is above a threshold that rises with the interest rate. The other claim that retained profits from the interest revenues of banks create an imperative is based on circuitist models that we consider refutable. Their accounting is inconsistent, and a modeling assumption central for a growth imperative is not underpinned theoretically: Bank's equity capital has to increase even if debt does not. This is a discrepancy between the authors' intentions in their texts and their actual models. We conclude that a monetary system based on interest-bearing debt-money with private banks does not lead to an 'inherent' growth imperative. If the stationary state is unstable, it is caused by agents' decisions, not by structural inevitableness.

4 From constrained optimization to constrained dynamics: extending analogies between economics and mechanics

Authors: Erhard Glötzl, Florentin Glötzl and Oliver Richters

Abstract: Economic equilibrium models have been inspired by analogies to stationary states in classical mechanics. To extend these mathematical analogies from constrained optimization to constrained dynamics, we formalize economic (constraint) forces and economic power in analogy to physical (constraint) forces and the reciprocal value of mass. Agents employ forces to change economic variables according to their desire and their power to assert their interest. These ex-ante forces are completed by constraint forces from unanticipated system constraints to yield the ex-post dynamics. The differential-algebraic equation framework seeks to overcome some restrictions inherent to the optimization approach and to provide an out-of-equilibrium foundation for general equilibrium models. We transform a static Edgeworth box exchange model into a dynamic model with procedural rationality (gradient climbing) and slow price adaptation, and discuss advantages, caveats, and possible extensions of the modeling framework.

5 Modeling economic forces, power relations, and stock-flow consistency: a general constrained dynamics approach

Authors: Oliver Richters and Erhard Glötzl

Abstract: We present a differential-algebraic equation framework of constrained dynamics to implement monetary Stock-Flow Consistent (SFC) models. Agents exert forces on the variables according to their desire, for instance to gradually improve their utility. The parameter 'economic power' corresponds to their ability to assert their interest. In analogy to Lagrangian mechanics, system constraints from accounting identities generate additional constraint forces that lead to unintended dynamics. We exemplify the procedure using a simple SFC model and reveal its implicit assumptions about power relations and agents' preferences.

6 Modeling the out-of-equilibrium dynamics of bounded rationality and economic constraints

Author: Oliver Richters

Modeling the out-of-equilibrium dynamics
of bounded rationality and economic constraints

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Abstract: The mathematical analogies between economics and classical mechanics can be extended from constrained optimization to constrained dynamics by formalizing economic (constraint) forces and economic power in analogy to physical (constraint) forces in Lagrangian mechanics. In a differential-algebraic equation framework, households, firms, banks, and the government employ forces to change economic variables according to their desire and their power to assert their interest. These ex-ante forces are completed by constraint forces from unanticipated system constraints to yield the ex-post dynamics. The out-of-equilibrium model combines Keynesian concepts such as the balance sheet approach and slow adaptation of prices and quantities with bounded rationality (gradient climbing) discussed in behavioral economics and agent-based models. Depending on the power relations and adaptation speeds, the model converges to a neoclassical equilibrium or not. The framework integrates different schools of thought and overcomes some restrictions inherent to optimization approaches, such as the problem of aggregating individual behavior into macroeconomic relations and the assumption of markets operating in or close to equilibrium.

Keywords: Simultaneous Equation Models; Stability of Equilibrium; Balance Sheet Approach; Constrained Dynamics; Out-of-equilibrium Dynamics, Lagrangian mechanics.

JEL: A12; B13; C30; C62; E10; E70.

1. Introduction

Dynamic economic models have to describe the time evolution of stocks, flows and other variables subject to economic constraints. Models based on general equilibrium, Keynesian disequilibrium or agent-based interaction differ in their assumptions about rationality, heterogeneity and adaptation speeds within the economy (Section 2.1). Introducing a novel out-of-equilibrium framework that tries to bridge some methodological gaps between these approaches, Glötzl et al. (2019) extend the historical analogies between general equilibrium models and Newtonian physics (Section 2.2): Similar to the forces of interacting 'bodies' under constraint from Lagrangian mechanics, the modeling approach depicts the economy from the perspective of economic forces and economic power. Economic force corresponds to the desire of agents to change certain variables, while economic power captures their ability to assert their interest to change them. Optimization is replaced by a gradient seeking approach in line with bounded rationality discussed in behavioral economics. The introduction of constraint forces, i. e. forces arising from system constraints, allows for a consistent assessment of ex-ante and ex-post dynamics of the dynamical system. The model presented in section 3 is based on a Keynesian balance sheet approach in which quantities adjust gradually and prices react slowly on supply-demand mismatches. It contains two households, two production sectors with input–output relations, banks and the government. A stability analysis reveals the conditions and power relations under which convergence to the usual neoclassical equilibrium is achieved. Section 5 concludes.

2. Modeling dynamics subject to constraints

2.1. Literature review

In general, a dynamic economic model is described by J agents and I variables $x_i(t)$ that can correspond to any stocks or flows of commodities, resources, financial liabilities, or any other variables or parameters such as prices or interest rates. The *structure* of the model consists of K economic constraints that remove many degrees of freedom. Constraints can be identities, relations "that hold by definition" (Allen, 1982, p. 4) such as the national income

account identity or accounting constraints in balance sheets. In material flow analysis (Brunner and Rechberger, 2004), constraints include laws of nature such as conservation of mass and energy as 'first laws' of chemistry and thermodynamics. Input–output relations or production functions imply certain technological limitations, while budget constraints are derived from the behavioral assumption that nobody is giving away money without an equivalent remuneration. The respect for identities is "the beginning of wisdom" in economics, but they must not be "misused to imply causation" (Tobin, 1995, p. 11). To derive causal arguments, a 'closure' has to be chosen that combines individual agency and the constraints: If the I variables were influenced by I behavioral equations, the system of equations would be overdetermined because of the additional K constraints. The schools of economic thought differ in their ways of making this system of equations solvable (Taylor, 1991), which will be discussed comparing (1) general equilibrium, (2) Keynesian disequilibrium and (3) agent-based models in the following.

In most general equilibrium models, each agent fully controls and voluntarily adapts all the stocks and flows directly affecting him (such as individual working hours or savings), resulting in various individual first-order conditions. Satisfying the K system constraints of market exchange can only be guaranteed by letting K prices adapt that make all the individual plans compatible with each other (neoclassical closure). Interacting via price signals, constraints imposed by other agents or system properties can be fully anticipated by the agents (Arrow and Hahn, 1971). The behavioral core of most Dynamic Stochastic General Equilibrium (DSGE) models is based on a representative agent with rational expectations that solves an intertemporal optimization problem subject to the constraints. The properties of utility and production functions, the Euler equation that describes the intertemporal trade-off, and the transversality condition as infinite time boundary condition guarantee that a unique and stable equilibrium path exists. External shocks combined with various frictions that slow down the return to equilibrium can create deviations from this optimum (Christiano et al., 2018; Lindé, 2018; Becker, 2008; Colander, 2009; Kamihigashi, 2008). While recent DSGE models also include some heterogeneity among households and firms (Kaplan et al., 2016; Christiano et al., 2018), many aspects of heterogeneity have to be left out. Galí (2018, p. 101) justifies this

with "tractability", but this is not only a question of complexity, but necessary to use this approach at all. Every optimization approach requires one single function to be optimized. Therefore, a society of utility maximizers has to be aggregated into a single social welfare function. Unfortunately, the assumptions made about individual rationality are "not enough to talk about social regularities", but it is necessary that "macro-level assumptions ... restrict the *distribution* of preferences or endowments" to guarantee a unique equilibrium (Rizvi, 1994, p. 359–63). Aggregation is possible if and only if demand is independent of the distribution of income among the agents (Gorman, 1961; Stoker, 1993; Kirman and Koch, 1986; Kirman, 1992), which Rizvi (1994, p. 363) calls an "extremely special situation". If agents had heterogeneous rates for discounting or intertemporal substitution, this condition would not be satisfied, and no unique stable equilibrium path would exist. These mathematical reasons restrict the integration of broader heterogeneity and social influences into DSGE models.

Keynesian disequilibrium models depart from the assumption that price adaptations can clear markets sufficiently fast. Departing from equilibrium assumptions implies that the *ex-ante* (planned) behavior does not necessarily respect the economic constraints. The *ex-post* (actual) dynamics are influenced by both system constraints and the agency of others. The quantities of demand or supply do not necessarily coincide, and terms such as "forced saving" or "involuntary unemployment" (Barro and Grossman, 1971) imply that agents cannot have complete control over the variables affecting them. For example, in some Keynesian disequilibrium models quantities of voluntary exchange are rationed by the 'short-side': Depending on market conditions, demand is limited by insufficient supply or otherwise (Benassy, 1975; Malinvaud, 1977). In contrast, some Post-Keynesian models consider the labor market to be purely demand-led and employees have no influence on working times: The K constraints that guarantee stock-flow consistency are satisfied by simply dropping K behavioral equations (Godley and Lavoie, 2012; Caverzasi and Godin, 2015). This one-sided 'drop closure' is justified if and only if exactly K stocks or flows are unaffected by agency, but only determined by the constraints (for a critique, see Richters and Glötzl, 2020).

Agent-based models (ABM) assume that individuals cannot solve infinitely dimensional opti-

mization problems, but use bounded rationality instead. Interactions between heterogeneous agents matter beyond market prices, and social interaction, social norms, power relations or institutions influence economic choices. Compared to selfish utility maximizers, this corresponds to a broader version of methodological individualism (Gallegati and Richiardi, 2009). ABM describe how quantities and prices can converge to a (statistical) equilibrium, but also discontinuities, tipping points, lock-ins or path dependencies can be studied (Kirman, 2010). The aggregate dynamics cannot be deduced from individual behavior that is often modeled as a sequence of simple rules. ABM lack a common core, and different coordinating mechanisms such as price adaptations, auctions, matching algorithms or quantity rationing are implemented to account for the economic constraints (Tesfatsion, 2006; Gintis, 2007; Gallegati and Richiardi, 2009; Ballot et al., 2014; Riccetti et al., 2015). Unfortunately, many ABM fail to actually satisfy stock-flow consistency: For example, in the exit-entry process of firms, defaulted firms are often simply recapitalized "ex-nihilo", violating economic identities and leading to logically incoherent flows and stocks evolutions. This lead to calls for stock-flow consistent agent-based modeling (Caiani et al., 2016; Caverzasi and Russo, 2018).

In the following, this paper presents a novel out-of-equilibrium framework that tries to bridge some methodological gaps between general equilibrium, disequilibrium and agent-based models. Compared to DSGE, the model goes back two steps and does without infinite intertemporal optimization and stochastic shocks, but removes the restriction that all utility functions can be aggregated into a social welfare function. It describes the interaction of bounded rational agents that exert economic forces to improve their situation (gradient climbing) subject to the economic constraints. The simultaneous processes of trade and price adaptation may dynamically converge towards equilibrium.

2.2. General Constrained Dynamics framework

The paper carries on an "unfinished business" of the early neoclassicals such as Irving Fisher or Vilfredo Pareto (Leijonhufvud, 2006, pp. 26–30): Inspired by the description of stationary states in classical mechanics, they derived an economic theory of static equilibrium (Pikler,

1955; Mirowski, 1989; Grattan-Guinness, 2010; Glötzl et al., 2019). Despite some efforts, they were unable to describe analoguously the adaptive *processes* that were thought to converge to the states analyzed in *static* theory (Donzelli, 1997; McLure and Samuels, 2001; Leijonhufvud, 2006).

Glötzl (2015) and Glötzl et al. (2019) took up this old challenge, introducing an economic framework inspired by the concept of constrained dynamics from Lagrangian mechanics (Lagrange, 1788; Flannery, 2011). In the General Constrained Dynamics framework, each agent seeks to change the existing configuration in the direction of his desires, but is subject to external constraints that can typically be written as:

$$0 = Z_k(x, \dot{x}), \quad k \in \{1, \dots K\}. \tag{1}$$

The dynamics of the model are the result of *economic forces* and *economic power*: An economic force f_{ji} corresponds to the desire of agent j to change a certain variable x_i. Economic power μ_{ji} captures the ability of an agent j to assert its interest to change variable x_i.[1] The total impact on the variable x_i is the product of economic force and power $\mu_{ji} f_{ji}$, i.e. the product of *desire* and *ability*:

$$\dot{x}_i = \sum_{j=1}^{J} \mu_{ji} f_{ji}(x). \tag{2}$$

All agents are unable to calculate infinite dimensional intertemporal optimization problems based on rational expectations about the reactions of the other market participants. Instead, they base their decisions on how much to work, invest, consume or save on the observation of current marginal utilities, profits, productivities and prices. They do not jump to the point of highest utility as rational utility *maximizers*, but instead try to 'climb up the utility hill' gradually by pushing the economy in the direction of highest marginal utility. In a continuous time framework, this can be modeled by defining the forces exerted by the agents as gradients

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[1]   The economic power factors μ_{ji} as 'ability to change' a variable correspond to the inverse of the mass in the Newtonian equations, in which mass is the 'resistance' to a change of velocity (Estola, 2017, p. 382; Glötzl et al., 2019).

of their utility functions. This corresponds to bounded rationality described by Lindenberg (2001, p. 248) as the "general desire to improve one's condition." With this gradient seeking approach, agents still satisfy the definition of rationality by Mankiw (2008, p. 6): "A rational decision maker takes an action if and only if the marginal benefit of the action exceeds the marginal cost." One might say that the agents in the economy are as rational as shortsighted first year economics students.

To guarantee consistency, Glötzl et al. (2019) proposed a 'Lagrangian closure' based on analogies to constraint forces in physics:[2] If all the variables x_i in a constraint Z_k are affected by agency, additional constraint forces z_{ki} are added to the time evolution of x_i, which together with the forces f_{ji} applied by all agents with power factors μ_{ji} creates the ex-post dynamics:

$$\dot{x}_i = \sum_{j=1}^{J} \mu_{ji} f_{ji}(x) + \sum_{k=1}^{K} z_{ki}(x, \dot{x}), \tag{3}$$

$$0 = Z_k(x, \dot{x}). \tag{4}$$

The constraint forces lead to unintended deviations of the actual time evolution from the planned one. In economics, the magnitude of the constraint forces z_{ki} cannot be derived from laws of nature, but reflect assumptions about adaptation processes within the economic system. In physics (Flannery, 2011; Glötzl et al., 2019), the time-dependent constraint forces z_{ki} can be calculated as

$$z_{ki}(x, \dot{x}) = \lambda_k \frac{\partial Z_k}{\partial x_i}, \tag{5}$$

or, if $\partial Z_k / \partial x_i \equiv 0$, as

$$z_{ki}(x, \dot{x}) = \lambda_k \frac{\partial Z_k}{\partial \dot{x}_i}. \tag{6}$$

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[2] The 'Newtonian Microeconomics' approach by Estola and Dannenberg (2012) and Estola (2017) is similar in the formalization of 'economic forces', but they accept that supply and demand differ not only ex-ante, but also ex-post (Estola, 2017, pp. 222, 386). This violation of economic identities occurs because they lack a formalization of economic constraint forces.

The additional variable λ_k ('Lagrangian multiplier') is introduced to make the model solvable. This rule from mechanics is a plausible choice also in economics (Glötzl et al., 2019), and the static version of these constraint forces is known from optimization exercises such as maximizing $U(x_1, x_2)$ subject to a budget constraint $0 = M - p_1 x_1 - p_2 x_2$. The first order condition $0 = \frac{\partial U}{\partial x_1} - \lambda p_1$ means that the 'utility force' and the constraint force cancel out, the latter given by the derivative of the constraint with a Lagrangian multiplier λ similar to Eqs. (5–6). The system of differential-algebraic equations (Eqs. 3–6) can be solved numerically for $x(t)$ and $\dot{x}(t)$.

Introducing this dynamic framework, Glötzl et al. (2019) presented a microeconomic Edgeworth box exchange model with two agents and two commodities and slow price adaptation that converges to the neoclassical contract curve for most parameters. Richters and Glötzl (2020) described a simple post-Keynesian stock-flow consistent disequilibrium model of the macroeconomic monetary circuit in this framework. This paper extends these ideas to a complex macroeconomic model.

3. The Model

3.1. Model structure: the constraints

The model studies the interaction of two households, two production sectors, a bank, and the government. They trade two consumer goods, labor and capital, financed by bank credit or equity. All agents show bounded rationality and try to increase their utility with a gradient climbing approach. Prices react slowly on demand–supply mismatches. As depicted in Fig. 1, the model consists of 42 economic variables:

- 11 financial balance sheet entries: M_a, M_b, V_a, V_b, E_{bank}, E_{f1}, E_{f2}, D_{f1}, D_{f2}, D_g, V_g,

- 4 stocks of real capital and inventories: K_{f1}, K_{f2}, S_{f1}, S_{f2},

- 8 prices: r_{f1}, r_{f2}, r_g, r_M, p_1, p_2, w_1, w_2,

- 6 flows of labor: L_{a1}, L_{a2}, L_{b1}, L_{b2}, L_{f1}, L_{f2},

- 8 flows of goods: C_{a1}, C_{a2}, C_{b1}, C_{b2}, G_{g1}, G_{g2}, A_{12}, A_{21},
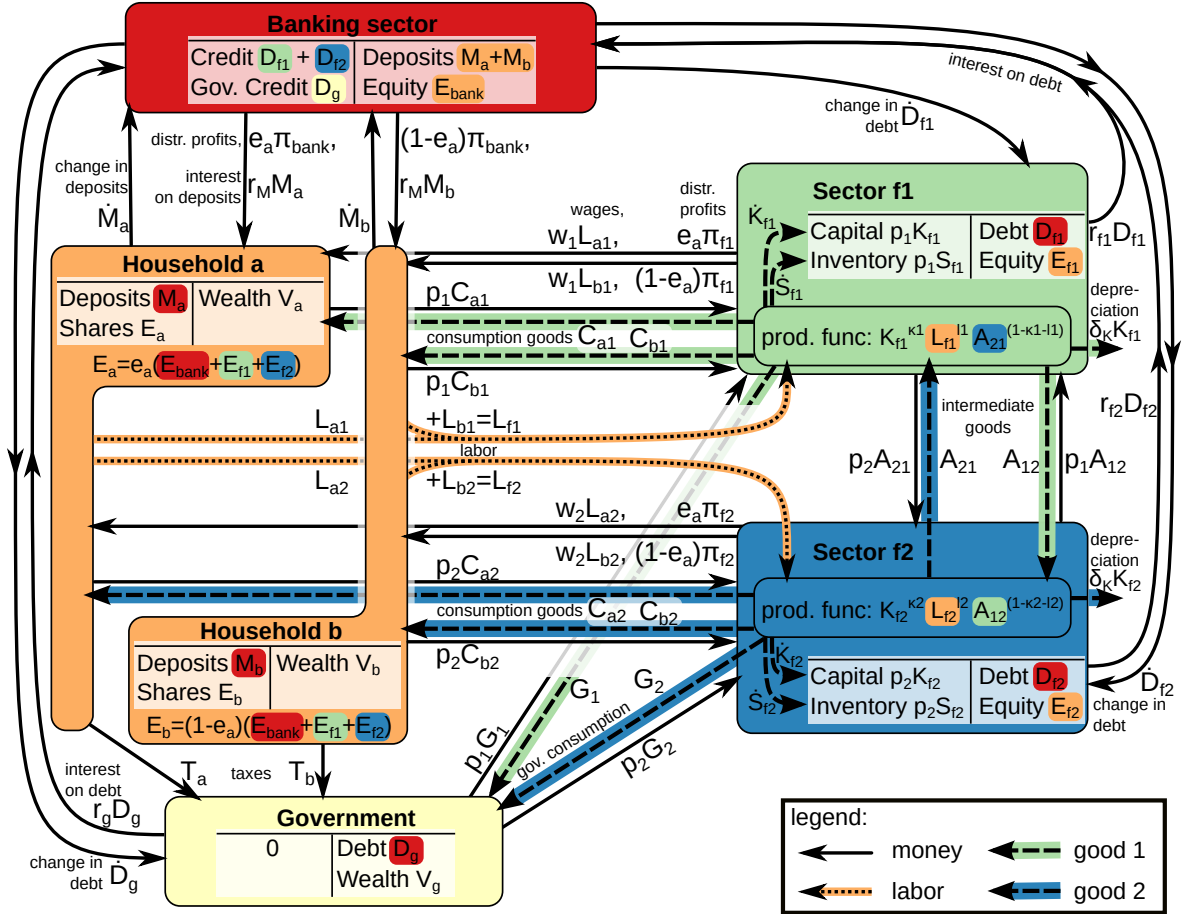
Figure 1: Model structure: The diagram depicts the balance sheets of the different sectors and the flows of money (black arrows) and goods (colored arrows) within the economy. The interconnectedness of the balance sheets is depicted by background colors: For example, the liability of sector $f1$ towards the bank has a red background, while the corresponding claim in the bank's balance sheet has a green background. Distinct flows that share an arrow are separated by commas.

The six balance sheets provide the constraints in Eqs. (7–12). Consistency of money flows provides the budget constraints (Eqs. 13–18). Eqs. (19–20) reflect consistency of labor flows, while consistency of good 1 and good 2 provides Eqs. (21–22).

- 5 flows of money: π_{f1}, π_{f2}, π_{bank}, T_a, T_b.

They are related by 16 constraints (Eqs. 7–22).

The consistency of double-entry bookkeeping in each of the six sectors provides six mathematical constraints:

$$0 = p_1(K_{f1} + S_{f1}) - D_{f1} - E_{f1}, \tag{7}$$

$$0 = p_2(K_{f2} + S_{f2}) - D_{f2} - E_{f2}, \tag{8}$$

$$0 = D_{f1} + D_{f2} + D_g - M_a - M_b - E_{bank}, \tag{9}$$

$$0 = M_a + e_a(E_{f1} + E_{f2} + E_{bank}) - V_a, \tag{10}$$

$$0 = M_b + (1 - e_a)(E_{f1} + E_{f2} + E_{bank}) - V_b, \tag{11}$$

$$0 = 0 - D_g - V_g. \tag{12}$$

The balance sheets are interconnected, because every financial claim has a corresponding liability, depicted by the colored background of the entries in Fig. 1. Household a holds a fraction e_a of the shares of the firm and banking sectors, while $1 - e_a$ is left for household b. They cannot trade their stakes in the firms. Eqs. (7–12) are used as definitions for E_{f1}, E_{f2}, E_{bank}, V_a, V_b and V_g. Therefore, no Lagrangian multipliers are needed to guarantee consistency. Summing all these equations yields $V_a + V_b + V_g = p_1(K_{f1} + S_{f1}) + p_2(K_{f2} + S_{f2})$, thus the actual wealth consists of real stocks of capital and inventories, because the debt relations cancel out. In the following, the equations for household b and sector $f2$ are provided, but explanations refer to household a and sector $f1$ only.

Six budget constraints track the flow of money for each agent. Household a consumes an amount C_{a1} at price p_1 from sector $f1$ and C_{a2} at price p_2 from sector $f2$. It works an amount L_{a1} for wage w_1 in sector $f1$ and L_{a2} for wage w_2 in sector $f2$, but has to pay taxes, for simplicity only on labor income, with an exogenous tax rate θ: Additional to wages, it receives a share e_a of the total distributed profits of firms and banks, while the deposits M_a earn him a

yearly interest of $r_M M_a$. The budget constraints are:

$$
Z_a = 0 = \dot{M}_a + p_1 C_{a1} + p_2 C_{a2} - (1 - \theta)(w_1 L_{a1} + w_2 L_{a2})
$$
$$
\ldots - e_a(\pi_{f1} + \pi_{f2} + \pi_{bank}) - r_M M_a, \tag{13}
$$

$$
Z_b = 0 = \dot{M}_b + p_1 C_{b1} + p_2 C_{b2} - (1 - \theta)(w_1 L_{b1} + w_2 L_{b2})
$$
$$
\ldots - (1 - e_a)(\pi_{f1} + \pi_{f2} + \pi_{bank}) - r_M M_b. \tag{14}
$$

The government g pays interest r_g on government debt D_g and buys goods from the two sectors G_{g1} and G_{g2} at price p_1 and p_2. It levies wage taxes with a constant tax rate θ, which results in the following budget constraint:

$$
Z_g = 0 = p_1 G_{g1} + p_2 G_{g2} - \theta(w_1 L_{a1} + w_2 L_{a2} + w_1 L_{b1} + w_2 L_{b2}) + r_g D_g - \dot{D}_g. \tag{15}
$$

Sector $f1$ (and equivalently $f2$) has to pay a wage w_1 per unit of work, a price p_2 for intermediate goods A_{21} used in production, and interest r_{f1} on debt D_{f1}. Money inflows arise from selling goods at price p_1 to households, the government, and sector $f2$. The difference between money inflows and outflows is distributed as profits π_{f1} or changes the stock of debt $\dot{D}_{f1}$, implying the following budget constraints:

$$
Z_{f1} = 0 = w_1 L_{f1} + p_2 A_{21} + r_{f1} D_{f1} - p_1(C_{a1} + C_{b1} + G_{g1} + A_{12}) + \pi_{f1} - \dot{D}_{f1}, \tag{16}
$$

$$
Z_{f2} = 0 = w_2 L_{f2} + p_1 A_{12} + r_{f2} D_{f2} - p_2(C_{a2} + C_{b2} + G_{g2} + A_{21}) + \pi_{f2} - \dot{D}_{f2}. \tag{17}
$$

The banking sector receives interest payments on credits and pays interest $r_M M_a$ and $r_M M_b$ to households. The difference between money inflows and outflows is distributed as profits π_{bank} or changes the stock of equity $\dot{E}_{bank}$, implying the following budget constraint:

$$
Z_{bank} = 0 = r_M(M_a + M_b) - r_{f1} D_{f1} - r_{f2} D_{f2} - r_g D_g + \pi_{bank} + \dot{E}_{bank}. \tag{18}
$$

Note that the constraints Z_a, Z_b, Z_g, Z_{f1}, Z_{f2} and Z_{bank} are linearly dependent with the

time derivative of Eq. (9) – as in every stock-flow consistent model, one budget constraint is redundant (Godley, 1999, p. 395). Consequently, Eq. (9) can be dropped and will just serve as an initial condition for $t = 0$, resulting in 15 linearly independent constraints.

Labor input L_{f1} of sector $f1$ has to be identical to the amount of work in this sector by households a and b, interrelating the variables of those agents:

$$Z_{L1} = 0 = L_{a1} + L_{b1} - L_{f1}, \tag{19}$$

$$Z_{L2} = 0 = L_{a2} + L_{b2} - L_{f2}. \tag{20}$$

As households and firms influence all these variables, these constraints cannot be treated as definitions for one variable. Consequently, constraint forces with Lagrangian multipliers λ_{L1} and λ_{L2} are added to the time evolution of these variables to ensure consistency. λ_{L1} is negative if the desired change in variables would lead to ex-ante excess supply for labor in sector $f1$. It will show up as constraint force in the time evolution of L_{a1}, L_{b1} and L_{f1} (Eqs. 25, 27, 47). (Note: the index $_i$ is identical for the Lagrangian multipliers λ_i and the corresponding constraints Z_i throughout the paper).

A constraint within sector $f1$ is that total production given by a Cobb-Douglas production function depending on capital K_{f1}, labor L_{f1} and intermediate input A_{21} has to be equal to consumption by households $C_{a1} + C_{b1}$, government consumption G_{g1}, deliveries to sector $f2$ as intermediate goods A_{12}, gross investment $\delta_K K_{f1} + \dot{K}_{f1}$ and change in inventory $\dot{S}_{f1}$. Sector $f2$ is constructed symmetrically, assuming a circular-horizontal production structure.

$$Z_{P1} = 0 = K_{f1}{}^{\kappa_1} L_{f1}{}^{l_1} A_{21}{}^{1-\kappa_1-l_1} - \dot{K}_{f1} - \delta_K K_{f1} - C_{a1} - C_{b1} - G_{g1} - \dot{S}_{f1} - A_{12}, \tag{21}$$

$$Z_{P2} = 0 = K_{f2}{}^{\kappa_2} L_{f2}{}^{l_2} A_{12}{}^{1-\kappa_2-l_2} - \dot{K}_{f2} - \delta_K K_{f2} - C_{a2} - C_{b2} - G_{g2} - \dot{S}_{f2} - A_{21}. \tag{22}$$

These identities will be guaranteed by the Lagrangian multipliers λ_{P1} and λ_{P2}.

3.2. Agents' behavior

Given 42 variables and 15 linearly independent constraints, only 27 behavioral equations could be chosen without the concept of Lagrangian closure. To show the flexibility of the framework, both the Lagrangian closure and the drop closure will be used for different variables. In the latter case, the behavior is implemented as an algebraic equation, not a differential equation. The model considers behavioral influences on 34 variables, which results in 49 equations for 42 variables. Therefore, 7 Lagrangian multipliers have to be added, one for each constraint in which all the variables are influenced by behavior. The following sections explain the constraints and behavioral assumptions in detail for households, government, firms and banks.

3.2.1. Households

The households are assumed to derive utility from consumption and leisure. In each variable, the utility functions U_a and U_b satisfy the Inada conditions:[3]

$$U_{a(t)} = C_{a1(t)}{}^{\alpha_{C1}} C_{a2(t)}{}^{\alpha_{C2}} + (1 - L_{a1(t)} - L_{a2(t)})^{\alpha_L} \,, \tag{23}$$

$$U_{b(t)} = C_{b1(t)}{}^{\beta_{C1}} C_{b2(t)}{}^{\beta_{C2}} + (1 - L_{b1(t)} - L_{b2(t)})^{\beta_L} \,. \tag{24}$$

Ex-post, households' decisions must be consistent with the budget constraints (Eqs. 13–14). The constraint forces are proportional to the Lagrangian multiplier λ_a times the derivative of the constraint Z_a with respect to the particular variable (see section 2.2).

For work L, the derivative of the budget constraint yields $\frac{\partial Z_a}{\partial L_{a1}} = -(1-\theta)w_1$, $\frac{\partial Z_a}{\partial L_{a2}} = -(1-\theta)w_2$. Additionally, the structural equations (19–20) for labor have to be satisfied. To avoid that total labor in a sector is different from the sum of work performed by the two households in this sector, an additional constraint force is added. Following the Lagrangian closure, the constraint forces are proportional to the derivative, $\frac{\partial Z_{L1}}{\partial L_{a1}} = \frac{\partial Z_{L2}}{\partial L_{a2}} = +1$ and $\frac{\partial Z_{L1}}{\partial L_{f1}} = -1$, which implies that all these variables are adjusted by the same amount. If

---

[3] U is strictly increasing, strictly concave, continuously differentiable and $U'(0) = \infty$ and $U'(\infty) = 0$ in every argument.

$\dot{L}_{a1} + \dot{L}_{b1} > \dot{L}_{f1}$ ex-ante, the constraint force reduces $\dot{L}_{a1}$ and $\dot{L}_{b1}$ while increasing $\dot{L}_{f1}$ until consistency is reached. Instead, a post-Keynesian economist may assume that firms' demand fully determines households' supply of labor, which illustrates that the choice of constraint forces reflects assumption about power relations within the economy. Taken together, the gradient forces from the utility function and the constraint forces yield the following time evolution:

$$\dot{L}_{a1(t)} = -\mu_{aL1} \cdot \alpha_L \left(1 - L_{a1(t)} - L_{a2(t)}\right)^{\alpha_L - 1} - \lambda_{a(t)} w_{1(t)}(1 - \theta) + \lambda_{L1(t)}, \tag{25}$$

$$\dot{L}_{a2(t)} = -\mu_{aL2} \cdot \alpha_L \left(1 - L_{a1(t)} - L_{a2(t)}\right)^{\alpha_L - 1} - \lambda_{a(t)} w_{2(t)}(1 - \theta) + \lambda_{L2(t)}, \tag{26}$$

$$\dot{L}_{b1(t)} = -\mu_{bL1} \cdot \beta_L \left(1 - L_{b1(t)} - L_{b2(t)}\right)^{\beta_L - 1} - \lambda_{b(t)} w_{1(t)}(1 - \theta) + \lambda_{L1(t)}, \tag{27}$$

$$\dot{L}_{b2(t)} = -\mu_{bL2} \cdot \beta_L \left(1 - L_{b1(t)} - L_{b2(t)}\right)^{\beta_L - 1} - \lambda_{b(t)} w_{2(t)}(1 - \theta) + \lambda_{L2(t)}. \tag{28}$$

For consumer goods, Eqs. (21–22) have to be satisfied, guaranteeing that goods produced are identical to those consumed, invested, stored or delivered to the other sector. Any ex-ante mismatch is compensated by adding constraint forces with factor $\frac{\partial Z_{P1}}{\partial C_{a1}} = \frac{\partial Z_{P2}}{\partial C_{a2}} = -1$ and Lagrangian multipliers λ_{P1} and λ_{P2} to the equation of motion. The derivative of the budget constraint yields $\frac{\partial Z_a}{\partial C_{a1}} = p_1$, $\frac{\partial Z_a}{\partial C_{a2}} = p_2$. The time evolution is given by:

$$\dot{C}_{a1(t)} = \mu_{aC1} \cdot \alpha_{C1} C_{a1(t)}^{\alpha_{C1}-1} C_{a2(t)}^{\alpha_{C2}} + \lambda_{a(t)} p_{1(t)} - \lambda_{P1(t)}, \tag{29}$$

$$\dot{C}_{a2(t)} = \mu_{aC2} \cdot \alpha_{C2} C_{a1(t)}^{\alpha_{C1}} C_{a2(t)}^{\alpha_{C2}-1} + \lambda_{a(t)} p_{2(t)} - \lambda_{P2(t)}, \tag{30}$$

$$\dot{C}_{b1(t)} = \mu_{bC1} \cdot \beta_{C1} C_{b1(t)}^{\beta_{C1}-1} C_{b2(t)}^{\beta_{C2}} + \lambda_{b(t)} p_{1(t)} - \lambda_{P1(t)}, \tag{31}$$

$$\dot{C}_{b2(t)} = \mu_{bC2} \cdot C_{b1(t)}^{\beta_{C1}} \beta_{C2} C_{b2(t)}^{\beta_{C2}-1} + \lambda_{b(t)} p_{2(t)} - \lambda_{P2(t)}. \tag{32}$$

An extension to 'positional' or 'conspicuous' consumption (Stiglitz, 2008; Dutt, 2009) could be modeled by adding a positive influence of household b on consumption decisions by household a.

The desired change in deposits held by households $\dot{M}_a$ and $\dot{M}_b$ reflects an intertemporal choice, but note that the bounded rational households cannot solve infinite optimization problems. We

assume that households value additional saving by the possible gain in leisure after a short period of time discounted by a factor ρ_a, at an average wage $(1 - \theta)(w_1 + w_2)/2$. Combining this behavioral force with with power factor μ_{aM} and the constraint force from the budget constraint with factor $\frac{\partial Z_a}{\partial M_a} = 1$ leads to:

$$\dot{M}_{a(t)} = \mu_{aM}(1 + \alpha_r(r_{M(t)} - \rho_a))\frac{2\alpha_L\left(1 - L_{a1(t)} - L_{a2(t)}\right)^{\alpha_L - 1}}{(1 - \theta)(w_{1(t)} + w_{2(t)})} + \lambda_{a(t)}, \tag{33}$$

$$\dot{M}_{b(t)} = \mu_{bM}(1 + \beta_r(r_{M(t)} - \rho_b))\frac{2\beta_L\left(1 - L_{b1(t)} - L_{b2(t)}\right)^{\beta_L - 1}}{(1 - \theta)(w_{1(t)} + w_{2(t)})} + \lambda_{b(t)}. \tag{34}$$

The parameter α_r captures how strongly household a considers this intertemporal choice. For an alternative specification with a simple 'money in the utility function' approach (Sidrauski, 1967), see Richters and Glötzl (2020).

3.2.2. Government

In this simple model, the government does not own assets or accumulates a stock of capital, but simply finances government consumption by tax income and debt. The government derives utility from buying goods and has a disutility that grows with government debt. The following utility function is chosen to illustrate that a utility function independent from the households choice can be specified, because there is no need to aggregate the individual utilities to a social welfare function before solving the model:

$$U_{g(t)} = G_{g1(t)}{}^{\gamma G1} + G_{g2(t)}{}^{\gamma G2} - \gamma_D(1 + \gamma_r r_{g(t)})\left(\frac{D_{g(t)}}{p_{1(t)} + p_{2(t)}}\right)^2. \tag{35}$$

Further assets and roles for the government such as redistribution, market stabilization or provision of public goods may be implemented in the future.

The government tries to improve its utility. Together with constraint forces proportional to $\frac{\partial Z_g}{\partial G_{g1}}$, $\frac{\partial Z_g}{\partial G_{g2}}$ and $\frac{\partial Z_g}{\partial D_g}$, this yields:

$$\dot{D}_{g(t)} = -\mu_{gD} \cdot \gamma_D(1 + \gamma_r r_{g(t)})\frac{2D_{g(t)}}{p_{1(t)} + p_{2(t)}} - \lambda_{g(t)}, \tag{36}$$

$$\dot{G}_{g1(t)} = \mu_{gG1} \cdot \gamma_{G1} G_{g1(t)}^{\gamma_{G1}-1} + \lambda_{g(t)} p_{1(t)} - \lambda_{P1(t)}, \tag{37}$$

$$\dot{G}_{g2(t)} = \mu_{gG2} \cdot \gamma_{G2} G_{g2(t)}^{\gamma_{G2}-1} + \lambda_{g(t)} p_{2(t)} - \lambda_{P2(t)}. \tag{38}$$

As in Eqs. (29–32) for households, the constraint forces λ_{P1} and λ_{P2} correspond to ex-ante mismatches of supply and demand for goods.

As discussed above, the government sets taxation as proportional to labor income:

$$0 = \theta \left(w_{1(t)} L_{a1(t)} + w_{2(t)} L_{a2(t)} \right) - T_{a(t)}, \tag{39}$$

$$0 = \theta \left(w_{1(t)} L_{b1(t)} + w_{2(t)} L_{b2(t)} \right) - T_{b(t)}. \tag{40}$$

These algebraic equations are equivalent to adding summands $(\theta(w_1 L_{a1} + w_2 L_{a2}) - T_a)^2$ and $(\theta(w_1 L_{b1} + w_2 L_{b2}) - T_b)^2$ to the utility function U_g, and this 'desire' being pursued with infinite power μ_{gT} (see Richters and Glötzl, 2020).

3.2.3. Firms

The firms in sector $f1$ hold inventories S_{f1} that act as a buffer stock against unexpected changes in demand. From a modeling perspective, these buffer stocks are important as they avoid the system of equations to become stiff and unsolvable.

Sector $f1$ produces consumption goods for households $C_{a1} + C_{b1}$ and the government G_{g1}, change in inventories $\dot{S}_{f1}$, intermediate goods A_{12} to be bought by sector $f2$, and gross investment consisting of replacement investment compensating depreciation $\delta_K K_{f1}$ and net investment $\dot{K}_{f1}$. For tractability, firms' production $P_{f1(t)}$ is given by a Cobb-Douglas function with production inputs capital K_{f1}, labor L_{f1} and intermediate goods A_{21}, see Eqs. (21–22)

The behavior of firms consists of an inventory and dividend policy, and the goal to increase their profits. The targeted ratio $s_{f1}^\top$ of inventories to expected sales (gross investment plus sales to consumers, government, and sector $f2$) is constant. The firms exert a force linearly increasing with the mismatch between targeted and actual inventories. Similar to Eqs. (29–32), a constraint force proportional to λ_{P1} with factor $\frac{\partial Z_{P1}}{\partial S_{f1}} = -1$ has to be added, assuming that

every part of demand will be negatively affected if ex-ante demand is bigger than ex-ante supply, to guarantee ex-post consistency.

$$\dot{S}_{f1(t)} = \mu_{fS1} \left(s_{f1}^\top \left(C_{a1(t)} + C_{b1(t)} + G_{g1(t)} + A_{12(t)} + \delta_K K_{f1(t)} + \dot{K}_{f1(t)} \right) - S_{f1(t)} \right) - \lambda_{P1(t)}$$

$$= \mu_{fS1} \left(s_{f1}^\top \left(K_{f1(t)}^{\kappa_1} L_{f1(t)}^{l_1} A_{21(t)}^{1-\kappa_1-l_1} - \dot{S}_{f1(t)} \right) - S_{f1(t)} \right) - \lambda_{P1(t)}, \tag{41}$$

$$\dot{S}_{f2(t)} = \mu_{fS2} \left(s_{f2}^\top \left(K_{f2(t)}^{\kappa_2} L_{f2(t)}^{l_2} A_{12(t)}^{1-\kappa_2-l_2} - \dot{S}_{f2(t)} \right) - S_{f2(t)} \right) - \lambda_{P2(t)}. \tag{42}$$

Concerning the production factors, firms exert forces as gradients of their expected profits as 'utility functions' U_{f1} and U_{f2}. Increasing production is costly not only because of direct inputs, but also because additional inventories according to Eqs. (41–42) have to be financed by credit:

$$U_{f1} = p_1 K_{f1}^{\kappa_1} L_{f1}^{l_1} A_{21}^{1-\kappa_1-l_1} - p_1 \delta_K K_{f1} - p_2 A_{21} - w_1 L_{f1}$$
$$\ldots - r_{f1} p_1 \left(K_{f1} + s_{f1}^\top \left(K_{f1}^{\kappa_1} L_{f1}^{l_1} A_{21}^{1-\kappa_1-l_1} - \dot{S}_{f1} \right) \right), \tag{43}$$

$$U_{f2} = p_2 K_{f2}^{\kappa_2} L_{f2}^{l_2} A_{12}^{1-\kappa_2-l_2} - p_2 \delta_K K_{f2} - p_1 A_{12} - w_2 L_{f2}$$
$$\ldots - r_{f2} p_2 \left(K_{f2} + s_{f2}^\top \left(K_{f2}^{\kappa_2} L_{f2}^{l_2} A_{12}^{1-\kappa_2-l_2} - \dot{S}_{f2} \right) \right). \tag{44}$$

Taking profits as basis for decision-making is similar to optimization approaches, but the difference is that firms do not jump directly to the point of highest profits by fully anticipating the reactions of households to changes in goods prices or wages. Instead, firms try to increase their profits using a gradient seeking approach, only fully aware of the current marginal productivities and prices without any expectation about future sales. The time evolution of the input factors consists of these profit driven forces and an additional constraint force with Lagrangian multiplier λ_{P1} to satisfy the production equations (21–22) ex-post.

For capital, the economic force exerted by the firms is given by $\mu_{fK1} \frac{\partial U_{f1}}{\partial K_{f1}}$, while the prefactor for the Lagrangian multiplier is calculated as $\frac{\partial Z_{P1}}{\partial K_{f1}}$:

$$\dot{K}_{f1(t)} = \mu_{fK1} \cdot p_{1(t)} \left(\left(1 - r_{f1(t)} s_{f1}^\top \right) \kappa_1 K_{f1(t)}^{\kappa_1-1} L_{f1(t)}^{l_1} A_{21(t)}^{1-\kappa_1-l_1} - \delta_K - r_{f1(t)} \right)$$
$$\ldots + \lambda_{P1(t)} \kappa_1 K_{f1(t)}^{\kappa_1-1} L_{f1(t)}^{l_1} A_{21(t)}^{1-\kappa_1-l_1}, \tag{45}$$

$$\dot{K}_{f2(t)} = \mu_{fK2} \cdot p_{2(t)} \left(\left(1 - r_{f2(t)} s_{f2}^\top \right) \kappa_2 K_{f2(t)}{}^{\kappa_2 - 1} L_{f2(t)}{}^{l_2} A_{12(t)}{}^{1 - \kappa_2 - l_2} - \delta_K - r_{f2(t)} \right)$$
$$\dots + \lambda_{P2(t)} \kappa_2 K_{f2(t)}{}^{\kappa_2 - 1} L_{f2(t)}{}^{l_2} A_{12(t)}{}^{1 - \kappa_2 - l_2}. \tag{46}$$

Note that total investment is given by $\dot{K}_{f1} + \delta_K K_{f1}$.

The time evolution of labor demand of firms contains an additional constraint force $-\lambda_{L1}$, added to guarantee consistency with labor supply by households according to Eqs. (19–20).

$$\dot{L}_{f1(t)} = \mu_{fL1} \left(p_{1(t)} \left(1 - r_{f1(t)} s_{f1}^\top \right) l_1 K_{f1(t)}{}^{\kappa_1} L_{f1(t)}{}^{l_1 - 1} A_{21(t)}{}^{1 - \kappa_1 - l_1} - w_{1(t)} \right)$$
$$\dots + \lambda_{P1(t)} l_1 K_{f1(t)}{}^{\kappa_1} L_{f1(t)}{}^{l_1 - 1} A_{21(t)}{}^{1 - \kappa_1 - l_1} - \lambda_{L1(t)}, \tag{47}$$

$$\dot{L}_{f2(t)} = \mu_{fL2} \left(p_{2(t)} \left(1 - r_{f2(t)} s_{f2}^\top \right) l_2 K_{f2(t)}{}^{\kappa_2} L_{f2(t)}{}^{l_2 - 1} A_{12(t)}{}^{1 - \kappa_2 - l_2} - w_{2(t)} \right)$$
$$\dots + \lambda_{P2(t)} l_2 K_{f2(t)}{}^{\kappa_2} L_{f2(t)}{}^{l_2 - 1} A_{12(t)}{}^{1 - \kappa_2 - l_2} - \lambda_{L2(t)}. \tag{48}$$

If labor is cheap compared to its contribution to production, the labor input is increased, but not instantaneously, and the constraint forces can lead to deviations from this plan.

For intermediate goods A_{21} produced by sector $f2$ and used by sector $f1$, the time evolution contains an additional term λ_{P2} with factor $\frac{\partial Z_{P2}}{\partial A_{21}} = -1$ because sector $f1$ is affected if there is insufficient production in sector $f2$:

$$\dot{A}_{21(t)} = \mu_{fA1} \left(p_{1(t)} \left(1 - r_{f1(t)} s_{f1}^\top \right) (1 - \kappa_1 - l_1) K_{f1(t)}{}^{\kappa_1} L_{f1(t)}{}^{l_1} A_{21(t)}{}^{-\kappa_1 - l_1} - p_{2(t)} \right)$$
$$\dots + \lambda_{P1(t)} (1 - \kappa_1 - l_1) K_{f1(t)}{}^{\kappa_1} L_{f1(t)}{}^{l_1} A_{21(t)}{}^{1 - \kappa_1 - l_1} - \lambda_{P2(t)}, \tag{49}$$

$$\dot{A}_{12(t)} = \mu_{fA2} \left(p_{2(t)} \left(1 - r_{f2(t)} s_{f2}^\top \right) (1 - \kappa_2 - l_2) K_{f2(t)}{}^{\kappa_2} L_{f2(t)}{}^{l_2} A_{12(t)}{}^{-\kappa_2 - l_2} - p_{1(t)} \right)$$
$$\dots + \lambda_{P2(t)} (1 - \kappa_2 - l_2) K_{f2(t)}{}^{\kappa_2} L_{f2(t)}{}^{l_2} A_{12(t)}{}^{1 - \kappa_2 - l_2} - \lambda_{P1(t)}. \tag{50}$$

The dividend policy is such that distributed profits π_{f1} and π_{f2} are total production minus input costs, which implies that changes in value of existing capital are not distributed:

$$0 = p_{1(t)} K_{f1(t)}{}^{\kappa_1} L_{f1(t)}{}^{l_1} A_{21(t)}{}^{1 - \kappa_1 - l_1} - p_{1(t)} \delta_K K_{f1(t)} - p_{2(t)} A_{21(t)} - w_{1(t)} L_{f1(t)}$$
$$\dots - r_{f1(t)} D_{f1(t)} - \pi_{f1(t)}, \tag{51}$$

$$0 = p_{2(t)} K_{f2(t)}{}^{\kappa_2} L_{f2(t)}{}^{l_2} A_{12(t)}{}^{1-\kappa_2-l_2} - p_{2(t)} \delta_K K_{f2(t)} - p_{1(t)} A_{12(t)} - w_{2(t)} L_{f2(t)}$$

$$\ldots - r_{f2(t)} D_{f2(t)} - \pi_{f2(t)}.$$

(52)

This is an example of a behavioral equation implemented as an algebraic equation, implying that π_{f1} and π_{f2} are *not* influenced by constraint forces. Alternatively, a principal–agent dilemma could be modeled by incorporating individual forces of shareholders trying to increase dividends while the management may favor retained earnings (La Porta et al., 2000). Using the accounting and budget constraints in Eqs. (7–8, 16–17), the time evolution of debt and equity can be calculated to be:

$$\dot{D}_{f1(t)} = p_{1(t)}(\dot{K}_{f1(t)} + \dot{S}_{f1(t)}),$$

(53)

$$\dot{D}_{f2(t)} = p_{2(t)}(\dot{K}_{f2(t)} + \dot{S}_{f2(t)}),$$

(54)

$$\dot{E}_{f1(t)} = \dot{p}_{1(t)}(K_{f1(t)} + S_{f1(t)}),$$

(55)

$$\dot{E}_{f2(t)} = \dot{p}_{2(t)}(K_{f2(t)} + S_{f2(t)}).$$

(56)

Thus new investment is financed by credit, while changes in value of existing capital changes the equity of firms: D_{f1}, D_{f2}, E_{f1} and E_{f1} adapt to satisfy the constraints and no Lagrangian multipliers λ_{f1} and λ_{f2} are necessary to guarantee consistency. Using these assumptions, there is no feedback from net worth on costs or volumes of external finance.

3.2.4. Banking sector

The balance sheet and budget constraint of the banking sector are Eqs. (9, 18). Banks are rather passive actors in this model: They lend money 'on demand' at the current interest rates r_{f1}, r_{f2} and r_g to firms and the government in line with the concepts of endogenous money creation (see Wray, 1990; Gross and Siebenbrunner, 2019). They pay interest $r_M M_a$ and $r_M M_b$ to households and distribute all their profits π_{bank} to the two households, here implemented as

an algebraic equation:

$$0 = r_{f1(t)}D_{f1(t)} + r_{f2(t)}D_{f2(t)} + r_{g(t)}D_{g(t)} - r_{M(t)}\left(M_{a(t)} + M_{b(t)}\right) - \pi_{bank(t)}. \tag{57}$$

A richer behavioral model of banks that includes credit rationing or agency costs may be integrated in the future.

3.2.5. Price development

The prices react to ex-ante mismatches between supply and demand. If the agents' plans would increase demand stronger than supply, the firms realize that they are unable to change their inventories as desired, which is the case if the Lagrangian multiplier $\lambda_{P1} > 0$. Sector $f1$ slowly increases the price p_1 with a linear reaction function to differences between the ex-ante values of supply and demand:

$$\dot{p}_{1(t)} = \mu_{p_1}\lambda_{P1(t)}, \tag{58}$$

$$\dot{p}_{2(t)} = \mu_{p_2}\lambda_{P2(t)}. \tag{59}$$

Similarly, the wages react on a mismatch between ex-ante supply and demand for labor:

$$\dot{w}_{1(t)} = \mu_w\lambda_{L1(t)}, \tag{60}$$

$$\dot{w}_{2(t)} = \mu_w\lambda_{L2(t)}. \tag{61}$$

The interest rates are adapted by the central bank according following a simple inflation targeting rule: If the average price change is above a target $\rho^\top$, interest rates are increased: As the cost for investment are proportional to the price level (Eqs. 53–54), this effectively increases the real interest rate for firms, thus the Taylor principle (Davig and Leeper, 2007) is satisfied.

$$\dot{r}_g = \dot{r}_{f1} = \dot{r}_{f2} = \dot{r}_M = \mu_r\left(\frac{\dot{p}_1/p_1 + \dot{p}_2/p_2}{2} - \rho^\top\right). \tag{62}$$

All the parameters μ reflect assumptions about power relations and adaptation speeds within the economy.

3.3. Time evolution and stationary states

The initial conditions have to satisfy the six balance sheet constraints (Eqs. 7–12), the two labor constraints (Eqs. 19–20) and the five algebraic equations for taxation (Eqs. 39–40) and profit distribution (Eqs. 51–52, 57). No further equilibrium conditions are presupposed.

As an example, Fig. 2 shows the time evolution for the initial conditions, power factors and further parameters summarized in Appendix A. At $t = 0$, plot (c) shows that for household b, the marginal utility of leisure divided by the wage $\frac{\partial U_b}{L_{b1}} \frac{1}{(1-\theta)w_1}$ is higher than the marginal utility of consuming good 1 divided by its price $\frac{\partial U_b}{C_{b1}} \frac{1}{p_1}$, and for good C_{b2} this value is even lower. Therefore, the forces of household b try to push the economy towards reducing work and consuming less, particularly of good 2. For household a, $\frac{\partial U_a}{C_{a2}} \frac{1}{p_2}$ is higher than $\frac{\partial U_a}{C_{a1}} \frac{1}{p_1}$, thus his forces try increase consumption of good C_{a2} compared to C_{a1}. Plot (f) compares the marginal productivities of inputs divided by their respective price. At $t = 0$, the marginal productivity of intermediate inputs A_{21} is high compared to the price p_2, while the marginal productivity of capital is lower than the interest rate. This is the reason why profits per equity π_{f1}/E_{f1} in sector $f1$ are very low, see plot (d). To improve profits, this sector exerts forces to increase A_{21} and to reduce K_{f1}. Sector $f2$ is in the opposite situation. The time evolution created by these ex-ante forces would not satisfy the constraints. For example, the changes in demand and supply for good 1 create a tendency of excess demand ($\lambda_{P1} > 0$). The corresponding constraint forces influence the dynamics such that the constraints are satisfied ex-post. Additionally, the price p_1 increases according to Eq. (58), while there is a tendency for excess supply for good 2, leading to a negative slope of p_2. The adjustment processes for quantities and prices ultimately converge to a stationary state whose properties can be calculated analytically.
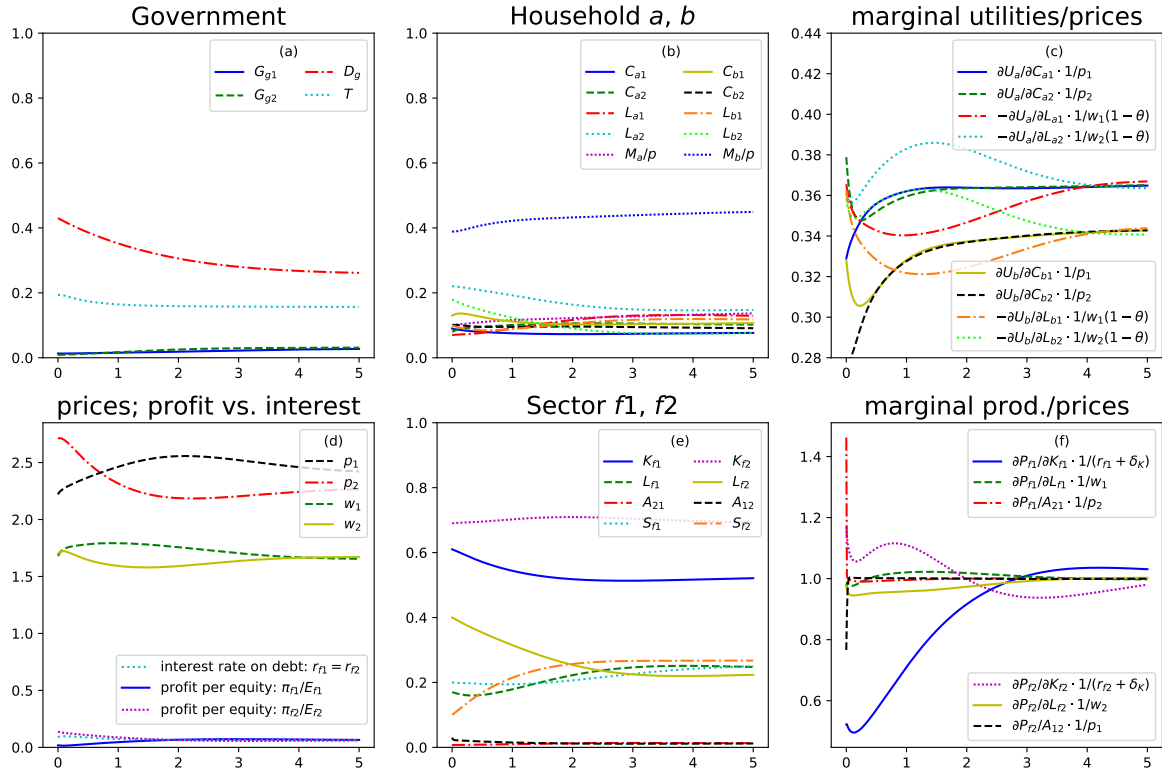
Figure 2: Plots (a), (b), (d) and (e) show the time evolution of the variables for different sectors. Plot (c) shows that for the two households, the marginal utilities for consumption and leisure, divided by their respective price, equalize over time. The budget equation constrains their choices, and the gradient climbing approach converges to the highest reachable level of utility. The same can be stated for plot (f) concerning the marginal input productivies for capital, labor and intermediate goods, divided by their price. Plot (d) shows that in equilibrium, profit paid per unit of equity is identical to the interest rate paid on debt.

4. Properties of the stationary state

The fixed point of a model is one with vanishing time derivatives in every variable. To derive the conditions for the stationary state, assume that every power factor is positive. From the price development (Eqs. 58–61), it follows that $\lambda_{P1} = \lambda_{P2} = 0$ and $\lambda_{L1} = \lambda_{L2} = 0$, thus in the stationary state, there is no mismatch between supply and demand for labor and goods.

For sector $f1$, the following conditions hold:

$$0 = p_1 \left(C_{a1} + C_{b1} + G_{g1} + A_{12} \right) - A_{21} p_2 - w_1 L_{f1} - r_{f1} D_{f1} - \pi_{f1} , \tag{63}$$

$$0 = s_{f1}^\top \left(C_{a1} + C_{b1} + G_{g1} + A_{12} + \delta_K K_{f1} \right) - S_{f1} , \tag{64}$$

$$0 = K_{f1}{}^{\kappa_1} L_{f1}{}^{l_1} A_{21}{}^{1-\kappa_1-l_1} - \delta_K K_{f1} - C_{a1} - C_{b1} - G_{g1} - A_{12} , \tag{65}$$

$$0 = (1 - r_{f1} s_{f1}^\top)\kappa_1 K_{f1}{}^{\kappa_1-1} L_{f1}{}^{l_1} A_{21}{}^{1-\kappa_1-l_1} - \delta_K - r_{f1} , \tag{66}$$

$$0 = p_1 (1 - r_{f1} s_{f1}^\top) l_1 K_{f1}{}^{\kappa_1} L_{f1}{}^{l_1-1} A_{21}{}^{1-\kappa_1-l_1} - w_1 , \tag{67}$$

$$0 = p_1 (1 - r_{f1} s_{f1}^\top)(1 - \kappa_1 - l_1) K_{f1}{}^{\kappa_1} L_{f1}{}^{l_1} A_{21}{}^{-\kappa_1-l_1} - p_2 . \tag{68}$$

This result is independent on the power factors. In this specification of the model, economic power influences the adaptation processes, but not the equilibrium reached. With an inventory target of $s_{f1}^\top = 0$, the factor share is identical to the output elasticity, the exponent of the production factor in the Cobb-Douglas function, as in neoclassical competitive equilibrium. With $s_{f1}^\top > 0$, a part of total income goes to interest payments related to inventory holding that do not contribute to increased production.

Using the definition (valid because $\dot{K}_{f1} = \dot{S}_{f1} = 0$)

$$P_{f1} = K_{f1}{}^{\kappa_1} L_{f1}{}^{l_1} A_{21}{}^{1-\kappa_1-l_1} = \delta_K K_{f1} + C_{a1} + C_{b1} + G_{g1} + A_{12} , \tag{69}$$

Eqs. (65–68) can be simplified to:

$$p_1 S_{f1} = p_1 s_{f1}^\top P_{f1} , \tag{70}$$

$$(r_{f1} + \delta_K)P_{f1}K_{f1} = p_1(1 - r_{f1}s_{f1}^\top)\kappa_1 P_{f1}, \tag{71}$$

$$p_2 A_{21} = p_1(1 - r_{f1}s_{f1}^\top)(1 - \kappa_1 - l_1)P_{f1}, \tag{72}$$

$$w_1 L_{f1} = p_1(1 - r_{f1}s_{f1}^\top)l_1 P_{f1}. \tag{73}$$

The debt is given by:

$$D_{f1} = p_1(K_{f1} + s_{f1}^\top P_{f1}) - E_{f1}. \tag{74}$$

Substituting these results in the definition of profit π_{f1} in Eq. (51) yields (see Appendix B):

$$\pi_{f1} = p_1(C_{a1} + C_{b1} + G_{g1} + A_{12}) - A_{21}p_2 - w_1 L_{f1} - r_{f1}D_{f1} = r_{f1}E_{f1}. \tag{75}$$

This derivation shows that in the stationary state, the profits are a compensation for equity capital E_{f1}, and both equity capital and credit have the same rate of return, see Fig. 2(d). This corresponds to the first theorem by Modigliani and Miller (1958), assuming that no financial frictions and no difference in riskiness exists.

For household a, we assume that $\mu_{aC1} = \mu_{aC2} = \mu_{aM} = \mu_{aL1} = \mu_{aL2}$, implying that households have the same power to influence all their variables. The following conditions hold:

$$0 = p_1 C_{a1} + p_2 C_{a2} - (1 - \theta)(w_1 L_{a1} + w_2 L_{a2}) - r_M M_a - e_a(\pi_{f1} + \pi_{f2} + \pi_{bank}), \tag{76}$$

$$0 = \alpha_{C1}(C_{a1})^{\alpha_{C1}-1}(C_{a2})^{\alpha_{C2}} + \lambda_a p_1, \tag{77}$$

$$0 = (C_{a1})^{\alpha_{C1}}\alpha_{C2}(C_{a2})^{\alpha_{C2}-1} + \lambda_a p_2, \tag{78}$$

$$0 = -\alpha_L(1 - L_{a1} - L_{a2})^{\alpha_L-1} - \lambda_a w_1(1 - \theta), \tag{79}$$

$$0 = -\alpha_L(1 - L_{a1} - L_{a2})^{\alpha_L-1} - \lambda_a w_2(1 - \theta), \tag{80}$$

$$0 = r_M - \rho_a. \tag{81}$$

The equations imply that total income from wages and capital is equal to taxes and consumption, and that the wages in both sectors have to be identical. Cancelling λ_a from all the equations

yields the first order conditions for consumers in general equilibrium models:

$$-\frac{\partial U_a/\partial L_{a1}}{(1-\theta)w_1} = -\frac{\partial U_a/\partial L_{a2}}{(1-\theta)w_2} = \frac{\partial U_a/\partial C_{a1}}{p_1} = \frac{\partial U_a/\partial C_{a2}}{p_2}.$$ (82)

The ratio of prices equals the ratio of marginal utilities, thus the utility from the last monetary unit spent on each good must be the same and identical to the disutility of increasing working time divided by the wage after tax $(1-\theta)w_1$. In equilibrium, the interest rate on deposits r_M equals the rate of time preference ρ_a. Note that this stationary state can be reached if and only if $\rho_a = \rho_b$, as r_M cannot converge to two distinct values simultaneously. If $\rho_a > \rho_b$, household a accumulates debt to finance consumption, as a no-ponzi condition is missing in this model. One way to relax this condition in the future would be to let the bank charge heterogeneous interest rates, depending on the debt-income ratio, which would allow the interest rate on debt for household a to rise to ρ_a.

The total income distributed from sector $f1$ to households a and b before taxation is given by (see Appendix B):

$$\pi_{f1} + r_{f1}D_{f1} + w_1L_{f1} = p_1P_{f1} - p_2A_{21} - \delta_K p_1 K_{f1}.$$ (83)

Total income is equal to production minus intermediate purchases minus depreciation.

For the government, the equations in the stationary state are, assuming $\mu_{gG1} = \mu_{gG2} = \mu_{gD}$:

$$0 = \gamma_{G1}G_{g1}{}^{\gamma_{G1}-1} + \lambda_g p_1,$$ (84)

$$0 = \gamma_{G_{g2}}G_{g2}{}^{\gamma_{G2}-1} + \lambda_g p_2,$$ (85)

$$0 = -2\gamma_D(1 + \gamma_r r_g)D_g/(p_1 + p_2) - \lambda_g,$$ (86)

$$0 = r_g D_g + p_1 G_{g1} + p_2 G_{g2} - T_a - T_b,$$ (87)

$$0 = \theta p_1(1 - r_{f1}s_{f1}^\top)l_1 P_{f1} + \theta p_2(1 - r_{f2}s_{f2}^\top)l_2 P_{f2} - T_a - T_b.$$ (88)

The stationary state is reached if tax income covers government expenditures and interest

payments on government debt D_g. The disutility of one additional unit of debt is identical to the utility gained by buying goods for this unit.

Overall, the stationary state satisfies all the condition usually presupposed in static neoclassical general equilibrium models.

4.1. Local and global stability

The differential-algebraic equation framework poses a challenge for the local stability analysis. Because of the constraints, the variables cannot be varied independently: A change in working hours necessarily implies a change in production, inventories, wage income, saving etc. The six balance sheet constraints (Eqs. 7–12), the two labor constraints (Eqs. 19–20) and the five algebraic equations for taxation (Eqs. 39–40) and profit distribution (Eqs. 51–52, 57) have to be guaranteed even after the shock. Additionally, interest rates have to march in lockstep, $\dot{r}_g = \dot{r}_{f1} = \dot{r}_{f2} = \dot{r}_M$, and $E_{bank} = 0$. These 17 restrictions have to be fulfilled, and I chose T_a, T_b, L_{f1}, L_{f2}, r_{f1}, r_{f2}, r_M, E_{f1}, E_{f2}, E_{bank}, V_a, V_b, D_g, V_g, π_1, π_2 and π_{bank} to be determined by constraints, while the remaining 25 values are varied: $x = \{K_{f1}, K_{f2}, L_{a1}, L_{a2}, L_{b1}, L_{b2}, C_{a1}, C_{a2}, C_{b1}, C_{b2}, G_{g1}, G_{g2}, r_g, w_1, w_2, p_1, p_2, S_{f1}, S_{f2}, M_a, M_b, D_{f1}, D_{f2}, A_{12}, A_{21}\}$. The production constraints (Eqs. 21–22) are not problematic because the change in inventories $\dot{S}_{f1}$, $\dot{S}_{f2}$ can absorb the shock. The time evolution $\dot{x} = T(x)$ around the equilibrium x_{eq} can be linearized with the (25×25) Jacobian matrix J_T of all the first-order partial derivatives:

$$J_T(x_{eq}) := \left(\frac{\partial T_i}{\partial x_j}(x_{eq}) \right)_{i,j=1,\dots,n} = \begin{pmatrix} \frac{\partial T_1}{\partial x_1}(x_{eq}) & \frac{\partial T_1}{\partial x_2}(x_{eq}) & \dots & \frac{\partial T_1}{\partial x_n}(x_{eq}) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial T_n}{\partial x_1}(x_{eq}) & \frac{\partial T_n}{\partial x_2}(x_{eq}) & \dots & \frac{\partial T_n}{\partial x_n}(x_{eq}) \end{pmatrix} \tag{89}$$

The Jacobian matrix contains the reaction of the economy to a shock in equilibrium, as illustrated in Fig. 3.

The relevant quantities for the first order stability of the stationary state are the eigenvalues of the Jacobian J_T. An analytical calculation shows that 0 is a double eigenvalue, thus $J_T v_i = 0$ with v_i being two corresponding linearly independent eigenvectors. The first eigenvector
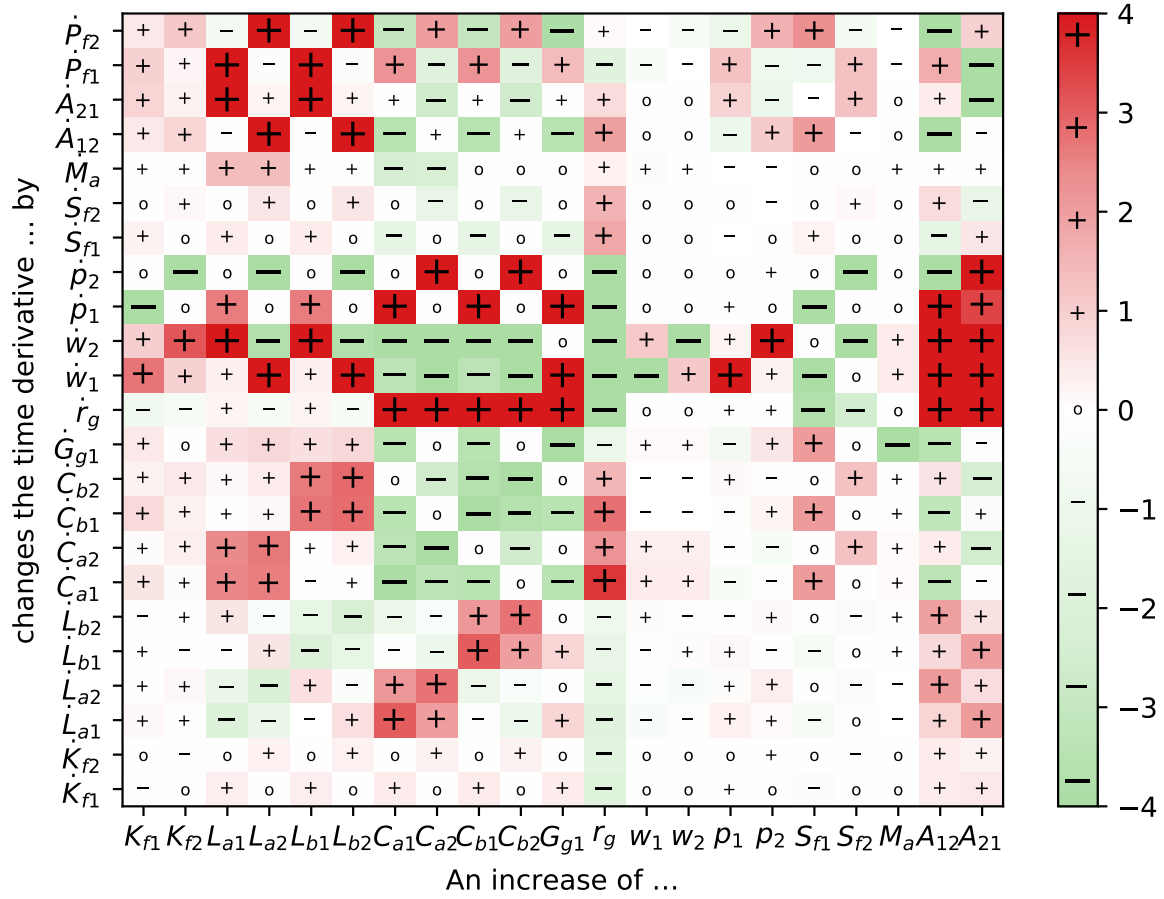
Figure 3: Selection of matrix entries of the Jacobian J_T in Eq. (89) illustrate the impact of a small increase in one of the variables on the time evolution of other. The reactions of the time derivatives to deviations from the equilibrium can be extracted from the diagram.

For example, the penultimate column implies that an increase in intermediate trade A_{12} from sector $f1$ to $f2$ leads to a reduction of the inventory stock ($\dot{S}_{f1} < 0$), which leads to increasing prices p_1 an increase in price $\dot{p}_1 > 0$, and increasing inputs $\dot{L}_{a1}, \dot{L}_{b1}, \dot{K}_{f1}, \dot{A}_{21}$ and a negative time evolution of the other sales $\dot{C}_{a1}, \dot{C}_{b1}$ and $\dot{G}_{g1}$. The additional input for sector $f2$ leads to a an increasing inventory stock $\dot{S}_{f2}$, a decreasing price $\dot{p}_2$ and a corresponding increase in households' demand. The other inputs L_{a2}, L_{b2}, K_{f2} grow, because the additional input A_{12} increases their marginal productivities. The additional demand for labor and capital leads to increasing wages and interest rates. These rising costs together with lowered prices p_2 will reverse this development in the following and push the economy back to equilibrium.

corresponds to an increase of L_{a1} and L_{b2} by ΔL, while L_{a2} and L_{b1} are reduced by the same amount: Household a works longer in sector $f1$, but shorter in sector $f2$, and household b inversely. The aggregated variables L_a, L_b, L_{f1} and L_{f2} remain unchanged. The second eigenvector corresponds to an increase of D_{f1} and E_{f2} by ΔD, accompanied by a decrease of D_{f2} and E_{f1} by the same amount. Sector $f1$ is now financed to a larger share by debt instead of equity, while it is the inverse for sector $f2$. Correspondingly, interest payments by sector $f1$ are increased while distributed profits are decreased and inversely for sector $f2$, keeping total equity and total firms' debt unchanged. In both cases, the stationary state is not unique but path dependent in some microscopic variables, but sectoral production, allocation, distribution and consumption remain unchanged.

The other eigenvalues depend on the parameters, particularly the power factors μ, as revealed by the stability analysis in Fig. 4. Starting from the parameters in section 3.3, each power factor related to quantities (such as μ_{aC1}, μ_{fK1}, μ_{gD} ...) is multiplied by a common factor $\mu_{quantities}$, while power factors related to prices (such as μ_w, μ_{p_1} ...) are multiplied by a factor μ_{prices}. In the red part on the right, the biggest real part of the eigenvalues is bigger than zero, implying local instability. For $\mu_{quantities}$ big, the quantities react so strongly for example on profit opportunities that the oscillations of the system become unstable. The stationary state in the orange part is locally stable, but the time evolution does not converge to the equilibrium. For example, if $\mu_{quantities} = 0$, the numerical solver aborts because no market forces prevent capital or labor from taking negative values, leading to an undefined value of the production function. In the green part, the time evolution converges to the equilibrium derived in section 4. In the blue part, the system did not converged to a stationary state at $t = 100$.

The reaction functions of the economic actors and the price adaptation cannot as such guarantee global stability. If quantity adjustments are fast, the model becomes unstable, because the bounded rational firms do not anticipate the reactions of the other market participants to their change in production. Instead, they react on supply–demand mismatches by adapting production and prices. If this reaction is very strong, it can lead to growing inventory oscillations as pioneered by Metzler (1941). Faster price adaptation can sometimes improve local stability.
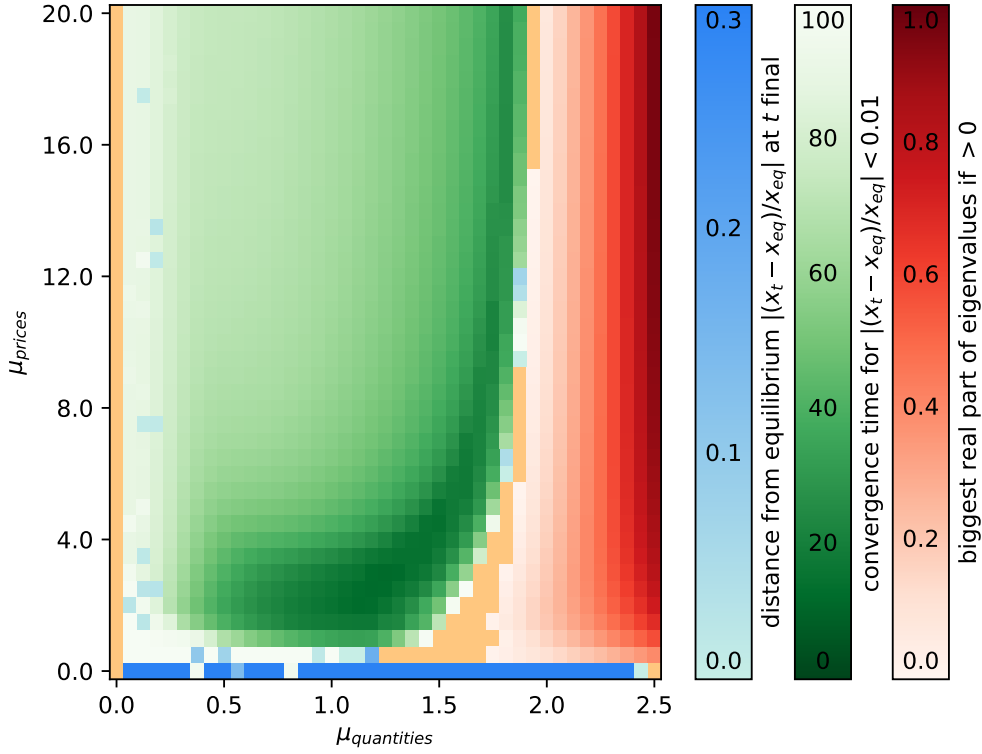
Figure 4: Stability analysis of the system, depending on the scaling factors μ_{prices} and $\mu_{quantities}$. The red color is used for combinations in which the biggest real part of the eigenvalues of the Jacobian J_T at the stationary state is above zero. Therefore, the system is locally unstable, the model shows explosive behavior and the numerical solver aborts. The orange color indicates the area where the real parts of all the eigenvalues are ≤ 0, but the numerical solver aborts nevertheless. This part of the system is locally stable, but shows no convergence for the initial conditions. The green part converges to the numerically determined equilibrium. The greener the color, the faster the convergence until $|(x_t - x_{eq})/x_{eq}| < 0.01$. In the blue part, the system did not converge to a stationary state at $t = 100$. The difference $|(x_t - x_{eq})/x_{eq}|$ at $t = 100$ is indicated by the blue color. For $\mu_{quantities} = 0$, the model is unstable because individual influences on quantities are negligible. For $\mu_{prices} = 0$ and $\mu_{quantities} > 0$, the model does not converge to a stationary state as the coordinating influence of price adaptation is missing.

Different from the way 'frictions' are commonly discussed in economic models as slowing down convergence to the equilibrium, very fast adaptations of prices and quantities make the equilibrium unattainable. In this model, intermediate adaptation speeds lead to the fastest convergence to equilibrium.

5. Discussion and Conclusions

This paper presented a dynamic modeling approach in continuous time that extends the analogies between mechanics and economics and depicts the economy from the perspective of economic forces and economic power. The conceptual model showed how General Constrained Dynamics can serve as a joint framework for general equilibrium, Keynesian disequilibrium and agent-based models: It includes some Keynesian features such as slow adaptation of prices and quantities or endogenous money creation. Similar to agent-based models, the heterogenous agents have bounded rationality, here modeled as utility improvement by 'gradient seeking'. Nevertheless, in the fixed points of the dynamical system, the first-order conditions of neoclassical general equilibrium solutions are satisfied and the power factors become irrelevant. The latter can be seen in light of the old debate whether control or economic laws determine market outcomes (Böhm-Bawerk, 1914). Different from DSGE models, fast adaptation of quantities and prices does not lead to fast convergence, but can amplify deviations from the equilibrium. As agents do not react optimally to changing conditions and do not anticipate the reaction of others, frictions have a stabilizing effect. It remains open whether this result holds if forward looking expectations of firms and the related intertemporal coordination problem are integrated. If this was the case, political regulation should concentrate on designing market frictions to stabilize markets, instead of eliminating them.

In the future, the model can be extended by additional forces to the time evolution: On the one hand, the approach allows to overcome some known restrictions of DSGE models, namely the aggregation problem, their assumption of rationality, and treatment of situations far from equilibrium. On the other hand, the above model contains many ad hoc assumptions which could

be refined depending on the application. This includes not only the principal–agent dilemma of dividend policies or credit rationing by banks, but also the influence of monopolists on prices. Furthermore, political economy issues such as the power relations and influences between politics and firms can be modeled, or households could mutually influence their decisions by 'positional' consumption. Stochastic shocks can be integrated, bearing in mind that the shocks have to satisfy the economic constraints. By the choice of the parameters that reflect 'economic power' in the sense of the ability to change certain variables and the integration of various social and market forces, the economic and social processes can be modeled in a flexible way.

In this paper, production and utility functions were chosen such that the dynamics converge to stable equilibria for most parameters. Economic models with multiple equilibria typically incorporate incomplete markets due to transaction costs or information asymmetries, increasing returns to scale, or market imperfections such as entry costs or external effects (Benhabib and Farmer, 1999). They were studied to explain issues such as asset bubbles, collateral shortages, liquidity dry-ups, bank runs, or financial crises (Miao, 2016). If multiple equilibria exist, a theory that describes the out-of-equilibrium dynamics is required to determine which of the equilibrium states is reached. A drawback of the GCD approach is that general equilibrium models with multiple markets are tremendously complex in the amount of variables that are simultaneously 'in equilibrium'. Consequently, providing models able to describe genuine out-of-equilibrium dynamics for all these variables poses a significant challenge. An intermediate approach could combine equilibrium dynamics with out-of-equilibrium processes where necessary. As the concept of Lagrangian closure draws on a mathematical similarity to static optimization models, the General Constrained Dynamics framework is a suitable candidate for this task.

Acknowledgements

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Appendix A   Initial conditions, power factors and parameters

Initial conditions: Household a: $L_{a1} = 0.07$; $L_{a2} = 0.22$; $C_{a1} = 0.09$; $C_{a2} = 0.08$; $M_a = 0.25$;
Household b: $L_{b1} = 0.10$; $L_{b2} = 0.18$; $C_{b1} = 0.13$; $C_{b2} = 0.10$; $M_b = 0.96$;
Firms: production inputs $K_{f1} = 0.61$; $K_{f2} = 0.69$; $L_{f1} = L_{a1} + L_{b1}$; $L_{f2} = L_{a2} + L_{b2}$;
$A_{12} = 0.03$; $A_{21} = 0.005$; inventories $S_{f1} = 0.20$; $S_{f2} = 0.10$; debt $D_{f1} = 0.52$; $D_{f2} = 0.26$;
equity $E_{f1} = p_1 (K_{f1} + S_{f1}) - D_{f1}$; $E_{f2} = p_2 (K_{f2} + S_{f2}) - D_{f2}$; $E_{bank} = 0$.
Government: expenditures $G_{g1} = 0.013$; $G_{g2} = 0.01$; debt $D_g = M_a + M_b - D_{f1} - D_{f2}$;
inflation target $\rho^\top = 0$;
Prices: $w_1 = 1.68$; $w_2 = 1.70$; $p_1 = 2.22$; $p_2 = 2.71$; $r_{f1} = r_{f2} = r_g = 0.09$; $r_M = 0.089$.

Power factors: Household a: $\mu_{aL1} = \mu_{aL2} = 1$; $\mu_{aC1} = \mu_{aC2} = 1$; $\mu_{aM} = 1$.
Household b: $\mu_{bL1} = \mu_{bL2} = 1$; $\mu_{bC1} = \mu_{bC2} = 1$; $\mu_{bM} = 1$.
Firms: $\mu_{fK1} = 0.5$; $\mu_{fK2} = 0.5$; $\mu_{fL1} = 1$; $\mu_{fL2} = 1$; $\mu_{fA1} = 1$; $\mu_{fA2} = 1$; $\mu_{fS1} = 1.5$; $\mu_{fS2} = 1$.
Government: $\mu_{gD} = 1$; $\mu_{gG1} = 1$; $\mu_{gG2} = 1$.
Price development: $\mu_{p1} = 2.5$; $\mu_{p2} = 2.5$; $\mu_w = 2$; $\mu_r = 1$.

Parameters: Household a: utility factors $\alpha_r = 2$; $\rho_a = 0.06$; $\alpha_L = 0.4$; $\alpha_{C1} = 0.25$; $\alpha_{C2} = 0.2$;
ownership share $e_a = 0.2$.
Household b: utility factors $\beta_r = 2$; $\rho_b = 0.06$; $\beta_L = 0.3$; $\beta_{C1} = 0.25$; $\beta_{C2} = 0.2$. ownership share
$1 - e_a$.
Firms: Cobb-Douglas exponents $\kappa_1 = 0.25$; $\kappa_2 = 0.3$; $l_1 = 0.7$; $l_2 = 0.55$; inventory to sales ratios
$s_{f1}^\top = 1$; $s_{f2}^\top = 1$; depreciation $\delta_K = 0.05$.
Government: utility factors $\gamma_{G1} = 0.6$; $\gamma_{G2} = 0.4$; $\gamma_D = 10$; $\gamma_r = 4$; tax rate $\theta = 0.2$.

Appendix B   Derivation of firms profits and households income in the stationary state

Substituting the results from section 4 into Eq. (51) yields:

$$\pi_{f1} = p_1 (C_{a1} + C_{b1} + G_{g1} + A_{12}) - A_{21} p_2 - w_1 L_{f1} - r_{f1} D_{f1} \tag{B.1}$$

$$= p_1 (P_{f1} - \delta_K K_{f1}) - A_{21} p_2 - w_1 L_{f1} - r_{f1} \left[p_1 (K_{f1} + s_{f1}^\top P_{f1})) - E_{f1} \right] \tag{B.2}$$

$$= p_1 P_{f1} \left[1 - (1 - r_{f1} s_{f1}^\top)(1 - \kappa_1) \right] - (\delta_K + r_{f1}) p_1 K_{f1} - r_{f1} p_1 s_{f1}^\top P_{f1} + r_{f1} E_{f1} \tag{B.3}$$

$$= p_1 P_{f1} \left[1 - (1 - r_{f1} s_{f1}^\top)(1 - \kappa_1) \right] - p_1 (1 - r_{f1} s_{f1}^\top)\kappa_1 P_{f1} - r_{f1} p_1 s_{f1}^\top P_{f1} + r_{f1} E_{f1} \tag{B.4}$$

$$= p_1 P_{f1} \left[1 - (1 - r_{f1} s_{f1}^\top)(1 - \kappa_1) - (1 - r_{f1} s_{f1}^\top)\kappa_1 - r_{f1} s_{f1}^\top \right] + r_{f1} E_{f1} \tag{B.5}$$

$$= r_{f1} E_{f1}. \tag{B.6}$$

Total income distributed to household a and b from sector $f1$ (interest via the banks) is:

$$\pi_{f1} + r_{f1} D_{f1} + w_1 L_{f1} \tag{B.7}$$

$$= r_{f1} p_1 (K_{f1} + s_{f1}^\top P_{f1}) + w_1 L_{f1} \tag{B.8}$$

$$= p_1 (1 - r_{f1} s_{f1}^\top)\kappa_1 P_{f1} - \delta_K p_1 K_{f1} + p_1 r_{f1} s_{f1}^\top P_{f1} + p_1 (1 - r_{f1} s_{f1}^\top)l_1 P_{f1} \tag{B.9}$$

$$= p_1 P_{f1} - (1 - \kappa_1 - l_1)p_1 P_{f1}(1 - r_{f1} s_{f1}^\top) - \delta_K p_1 K_{f1} \tag{B.10}$$

$$= p_1 P_{f1} - p_2 A_{21} - \delta_K p_1 K_{f1}. \tag{B.11}$$

Declarations of Authors' Contributions and Academic Integrity

Author contributions

Growth imperatives: Substantiating a contested concept.

This paper by Oliver Richters and Andreas Siemoneit was jointly written and revised over a period of more than three years. Both authors contributed to the literature review and to the creation of the figures, derived new definitions for key terms such as 'social coercion' and 'growth imperative', and applied them to a variety of economic and sociological theories. These personal contributions have been discussed, refined or discarded in countless telephone calls and face-to-face meetings. It is impossible to attribute individual parts or paragraphs to one of the authors.

Consistency and Stability Analysis of Models of a Monetary Growth Imperative.

This paper by Oliver Richters and Andreas Siemoneit was first a chapter of the above paper on growth imperatives, but was later extended into a separate work. The above description of the work process applies also here. Both authors contributed equally to the literature review in section 1. Both authors described and interpreted the results in section 2, performed the consistency analysis in section 3, and wrote the final section on results and conclusions. OR performed the stability analyses in the appendices.

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EG conceived the presented idea. FG and OR jointly wrote the literature review in section 2. EG, FG and OR formulated, described and analyzed the model. EG and OR programmed the model in Mathematica, OR programmed the model in python3. EG, FG and OR jointly wrote the discussion and conclusions.

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Declaration of Academic Integrity

Hiermit erkläre ich, dass ich die Ordnung über die Grundsätze zur Sicherung guter wissenschaftlicher Praxis an der Carl von Ossietzky Universität Oldenburg befolgt habe.

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Oldenburg, 3. Juli 2020

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